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My Parent Taught Me This Way: The European/Latino Multidigit Subtraction Algorithm and Confusions with a U.S. Algorithm

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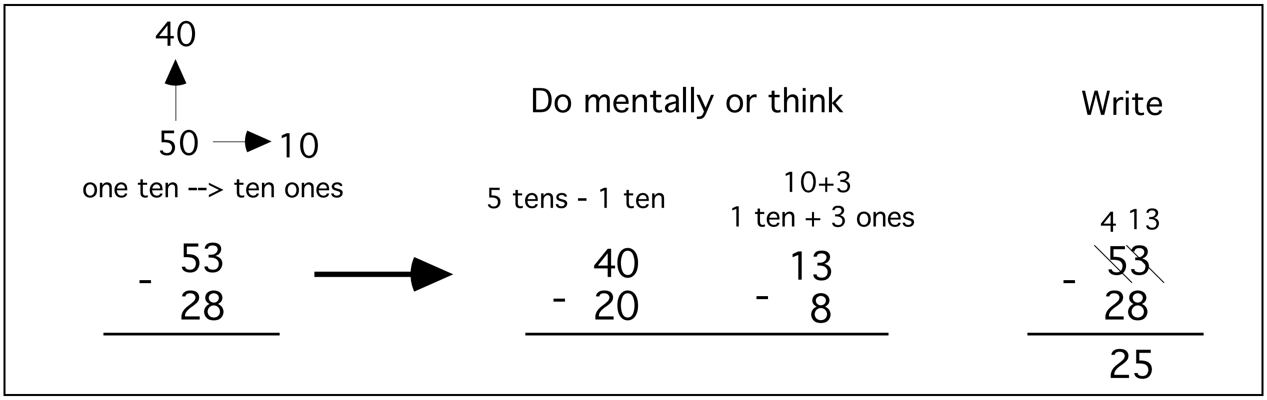
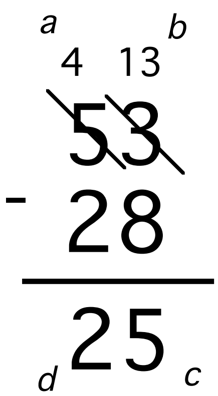
I remember the first time I saw subtraction done using the algorithm taught in many American schools. I was reading a paper on the typical mistakes children make when learning multidigit subtraction, and I felt like one of those children trying to make sense of the numbers in front of me. I wondered what those crossed-out numbers were and what the author of the paper meant by “ignored borrows.” I had to read the paper three times before I could figure out that my way of doing subtraction was very different from that of the author of the paper and of the children making those typical mistakes. There I was, a reasonably intelligent adult woman, and yet I could only make sense of the American subtraction algorithm with the aid of a paper. Since then I have met many other people who have shown an equal lack of understanding when faced with multidigit subtraction solved by American-schooled children. Those people and I did not necessarily share a language or a culture, but we did share a common trait: we had all been schooled in Europe or in South America.

I have spent the last three years working on a math (K-3) research project in bilingual inner-city schools where 88% of the children have a Latino background. 100% of children in the Spanish-speaking classrooms have at least one parent born in a Latino country (mostly Mexico) and over 50% of the children were actually born in Mexico or Puerto Rico. Of the English-speaking classes, 74% have at least one parent born in a Latino country (mostly Puerto Rico) and over 50% of the children were actually born in Mexico or Puerto Rico (Fuson and Feingold, 1996). Our approach is to allow children to use different methods with the support of ten-structured materials for understanding. However, we have worked with teachers who teach a common U.S. algorithm and have seen the confusions that result when parents from Europe or from Spanish-speaking countries in this hemisphere teach their child their “add-tens-to-both” method. Because parents will always try to help their children with math as they know it, it is helpful for teachers to understand the European/Latino algorithm, the errors that the children make with it, and the confusions that arise from mixing the U.S. and the European/Latino algorithm.

The Algorithms

A common U.S. algorithm: Borrow one ten to go with the ones (regroup the top number)

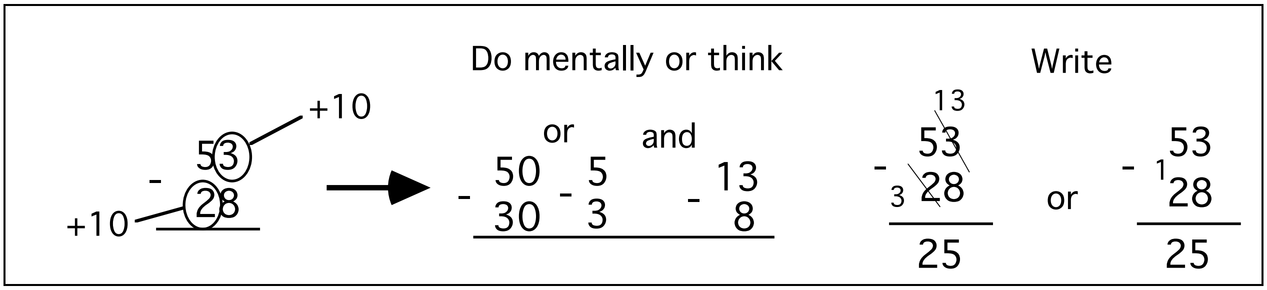




Borrow step:
I can’t take 8 away from 3, so I borrow 1 ten from the 5 tens. The 5 then becomes a 4 (a) and the 3 ones become 13 ones (b).

Subtract step:
13 take away 8 is 5 (c) and 4 tens take away 2 tens is 2 tens (d). The answer is 25.

The European/Latino algorithm: Add-a-ten-to-both-numbers



In this algorithm you change both numbers equally by adding a ten to each. However, you add one ten to the ones of the top number and the other ten to the tens of the bottom number. Also the actual subtracting is usually done as counting up. You do not think “13 take away 8 is 5” (13-8 = ?) but rather “from 8 to 13 is 5” (8 + ? = 13). The adding of a ten to the bottom number is expressed by the wording “I carry one”. This is the same wording used in the addition algorithm when a 1 (actually a ten) is carried over. For the European/Latino algorithm this opens the door to potential confusion of addition and subtraction.

$$\begin{array}{r} 5\overset{13}{\cancel{3}}\;^{a} \\ -\;{}_{c}\,3\,\cancel{2}8 \\ \hline {}_{d}\;25\;{}_{b} \end{array}$$

Add ten to top ones:

I can’t take 8 away from 3, so I make the 3 into a 13 (a).

Subtract the ones:

From 8 to 13 is 5 (b)

Add ten to bottom tens:

I carry one. 2 and 1 (the one I carry) is 3 (c).

Subtract the tens:

From 3 to 5 is 2 (d). The answer is 25.”

Typically one says thinks of writing/putting a 1 in 13 and carrying 1 in the tens rather than saying/thinking of adding tens. As with the U.S. algorithm, most parents do not understand why it works, or that you are really adding a ten to each multidigit number. They think as outlined above using rote steps.

Although we marked the changes on the numbers (the 3 ones and 2 tens are crossed out and changed into a 13 and a 3), in many cases those changes are made mentally and no record of them appears on the paper.

Typical Errors and Conflicts

Both algorithms will appear in a classroom even if a teacher does not teach them because they will come from the homes. Because some parents emphasize working mentally, teachers may not see marks to show the errors on a paper. Asking children to explain their methods can help the teacher uncover the reasons for the errors that do arise.

Partial confusion of addition and subtraction

The child correctly solves the “ones” subtraction, then carries one but adds all the numbers in the ten columns. The source of this mistake is easily understood if we recall that in the European/Latino algorithm you do in fact carry one to the ten column that you then add to the number to be subtracted. So the act of “carrying one” can shift a child to the addition algorithm, so s/he then adds everything. The child usually does not realize that s/he is mixing addition and subtraction, and just pointing that fact to them may not be enough. The phrase “My parent taught me this way.” is often heard in the classroom as an explanation for a procedure that they realize was not taught in class. Children must be helped to understand that in the European/Latino subtraction algorithm the 1 ten is added to the bottom number before they can go on with subtraction (unlike addition where the 1 carried over can be added to any of the numbers in the tens column, or even at the end to the total number of tens). Children who want to use this method might initially find it easier to “fix the problem for subtraction” first by adding the tens to the top and bottom number, recording as they do so. Then they could subtract. Changing the “I carry a one” language to “I am adding one (ten) to the bottom” also helps.

European/Latino error: Write 1 in the top ones and top tens so switch to addition in the tens column (1’s may only be done mentally and not written)

$$\begin{array}{r} {}^{b}1 \\ 5{}^{1}3 \\ -\ 28 \\ \hline {}_{c}85_{a} \end{array}$$

<u>Subtract ones:</u>
I can’t take 8 away from 3, so from 8 to 13 is 5 (a)
<u>Add tens:</u>
I carry one (b). 5 and 2 is 7 and 1 is 8 (c).

Some children begin with the U.S. borrowing algorithm, then switch algorithms during the move to the tens. They “carry one” but in doing so shift to adding as in the error above.

Mixed US-Latino version: Ungroup correctly, subtract ones, then add tens (1 and 4 and 13 may or may not be written)

$$\begin{array}{r} {}^{e}1 \\ {}^{b}4\ \ 13^{c} \\ {}_{a}\cancel{5}\cancel{3} \\ -\ 28 \\ \hline {}_{f}75_{d} \end{array}$$

(U.S.) <u>Borrow to subtract ones:</u>
I can’t take 8 away from 3, so I borrow 1 from the 5 (a). The 5 then becomes a 4 (b) and the 3 becomes a 13 (c).
(U.S.) <u>Subtract ones:</u>
13 take away 8 is 5 (d).
(E/L) <u>Carry the one:</u>
I carry one (e)
<u>E/L error: Shift to addition so add 1 to top instead to bottom and add everything:</u>
The 1 I carry (e) and 4 is 5 and 2 is 7 (f).

Mixed US-Latino version: Ungroup correctly, subtract ones, add 1 to top not bottom and subtract tens (1, 4 , 5 and 13 may or may not be written)

$$\begin{array}{r} e\ {}_{1}5\ f \\ b\ \cancel{4}\ \ 13\ c \\ a\ \cancel{5}\cancel{3} \\ -\ 28 \\ \hline g\ 35\ d \end{array}$$

(U.S.) <u>Borrow to subtract ones:</u>
I can't take 8 away from 3, so I borrow 1 from the 5 (a). The 5 then becomes a 4 (b) and the 3 becomes a 13 (c).
(U.S.) <u>Subtract ones:</u>
13 take away 8 is 5 (d).
(E/L) <u>Carry the one:</u>
I carry one (e)
<u>E/L error: Shift to addition so add 1 to top instead to bottom but then subtract:</u>
The 1 I carry and 4 is 5 (f). 5 take away 2 is 3 (g)
The answer is 35."

The result of this error is the same as if the child had forgotten to do the borrow when in fact s/he has not forgotten. If the child does the procedure mentally, the teacher has no way of knowing if the child forgot to do the borrow, did the borrow and then ignored it, or did something else (as would be the case here). A correction like "you forgot to borrow" would not be of use here because the child did not forget. A teacher needs to ask the child how s/he did it to discover the source of the error.

A final mistake that is not really so common comes from children that correctly do both algorithms at the same time. In this case children do both the borrow and the carry over correctly but their answer is one ten too small because they subtracted one ten twice.

$$\begin{array}{r} b\ 4\ \ 13\ c \\ a\ \cancel{5}\cancel{3} \\ -\ e\ 3\cancel{2}8 \\ \hline f\ \ 15\ d \end{array}$$

(U.S.) <u>Borrow to subtract ones:</u>
I can't take 8 away from 3, so I borrow 1 from the 5 (a). The 5 then becomes a 4 (b) and the 3 becomes a 13 (c).
(U.S.) <u>Subtract ones:</u>
13 take away 8 is 5 (d).
(E/L) <u>Add to bottom tens:</u>
I carry one, 2 and 1 is 3 (e)
(E/L) <u>Subtract tens:</u>
From 3 to 4 (or 5 take away 4) is 1 (f)

Although all the above errors are certainly possible, and to some degree common, among lower-achieving European/Latino children in American schools, the typical progression is for them to abandon the algorithm they were taught at home and to become proficient with the one taught at school, if a standard algorithm is taught. But some children remain confused for a long time. A better understanding on the part of both teachers and parents of the source of the confusions these children have could and should shorten the time in which these children lack a correct algorithm for multidigit subtraction. Teaching meaningfully using tens and ones language and drawings of tens and ones (see Fuson, Smith, Lo Cicero, 1997, 2002) and asking children to explain their thinking is crucial (Hufferd-Ackles, Fuson, & Sherin, 2004, 2015). Also a focus on the nature of the answer can be helpful because each error above has a particular pattern of variation from the correct answer.

Conclusions

In a classroom where math is taught with an emphasis on understanding, the U.S. algorithm shows a clear advantage over the European/Latino algorithm because it can be explained meaningfully. A child can be helped to understand that the borrow is actually a regrouping of 53 (5 tens and 3 ones) into 40+13 (4 tens and 13 ones). In the current world-wide trend of teaching math for understanding, the U.S. algorithm may start being taught in European and Latino schools. In some teacher colleges in Spain, the American algorithm is already being taught to would-be-teachers as a preferred alternative to the traditional one because it can be explained meaningfully (Salvador Llinares, personal communication). When and if that happens, teachers from those countries will benefit from understanding the two different algorithms and the errors associated with the unexpected mixing of both algorithms that come when children get conflicting messages at school and at home.

In a country like the United States where the student population can be so diverse, it is important that teachers find out what knowledge students from all backgrounds bring from their cultures and their homes and <u>accept it</u> rather than try to impose ours. Understanding how other algorithms work can provide the teacher with the necessary resources to uncover the reason behind certain mistakes that at first glance are unexplainable. Allowing alternative algorithms into the classroom would not only give voice to the knowledge coming from the homes, but also raises important mathematical issues that can be pursued. For example, children might be asked, “For what kinds of other problems might the general strategy of the European/Latino algorithm simplify your work?” (e.g. 437-199 could become 438-200 by adding 1 to both numbers).

Teachers can and should use the cultural richness that is often found in their classroom. Algorithms are by no means the only rich source of cultural differences, nor is multidigit subtraction the only algorithm that shows clear differences. The U.S. division algorithm is also noticeably different from the algorithm used in Latino countries as shown below. Understanding and openly accepting these differences can only be beneficial for students who might otherwise struggle between what is taught at home and at school.

U.S. division algorithm	Latino division algorithm
$\begin{array}{r} 36 \\ 24\overline{)879} \\ \underline{72} \\ 159 \\ \underline{144} \\ 15 \end{array}$	$\begin{array}{r\|l} 879 & 24 \\ \cline{2-2} 159 & 36 \\ 15 & \end{array}$ More steps are done mentally in this method, and the divisor is written to the right

For children who move from countries already having learned different algorithms in their schools, it is important that they be allowed to continue to use them. Many older children talk about being confused for years if they are made to switch algorithms. Instead, they can share their method with the class. Understanding how a new method works can be a fruitful math project for the whole class. Children could even become “algorithm detectives” who try to find and explain algorithms they find from interviewing relatives or people or from reading books. In these ways, math from other cultures can facilitate children’s mathematical understanding of underlying mathematical features and strategies and expand their appreciation of different ways to approach and record numerical thinking.

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