Korean Children's Understanding of Multidigit Addition and Subtraction

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FUSON, KAREN C., and KWON, YOUNGSHIM. *Korean Children's Understanding of Multidigit Addition and Subtraction.* CHILD DEVELOPMENT, 1992, **63**, 491–506. This study examined Korean second and third graders' understanding of multidigit addition and subtraction and particularly their ability to explain the trading required when a column sum of the addends is 10 or more. 72 middle-class second- and third-grade children (aged 7-4 to 8-4 and 8-4 to 9-4, respectively, at the time of the midyear interview) attending 2 schools in Seoul, Korea, were asked to solve 2- and 3-digit problems given in vertical form and then were individually interviewed about their conceptual understanding of such problems. Even though the second graders had not yet received instruction in school on 3-digit problems, children in both grades were quite accurate solvers of the multidigit addition and subtraction problems and demonstrated knowledge of the place-value names "ten" and "hundred." Every child also correctly identified the trade between the ones and tens columns as a traded ten. Most of the third graders identified the 1 written in the hundreds column on the addition problems as a hundred, but half of the second graders identified it as a ten. Most of the third graders also gave correct descriptions of the trading (regrouping, borrowing) required by a 3-digit subtraction problem with 2 zeros in the top number. Children used 3 different conceptual structures in discussing the already-solved problems: a multiunit quantities structure, a regular one/ten trades structure, and a combination of these two. These results are compared to the literature on the performance and conceptual structures of children in the United States.

Korean elementary school children aged 6, 7, and 8 carry out 2- and 3-digit addition and subtraction considerably more accurately than do their U.S. age-mates (Song & Ginsburg, 1987). Children in the United States experience considerable difficulty carrying out multidigit addition and subtraction requiring trades (regrouping, carrying/borrowing) between columns and in learning place-value concepts. For example, fewer than half of the third graders in the National Assessment of Educational Progress identified the hundreds digit correctly (Kouba et al., 1988); and U.S. children show other inadequacies in place-value understanding (Ginsburg, 1977; C. Kamii, 1985, 1986; M. Kamii, 1981; Labinowicz, 1985; Ross, 1986). Half of the third graders in the National Assessment of Educational Progress gave incorrect answers on a 3-digit subtraction problem requiring trading (Kouba et al., 1988), and children have particular difficulty with problems having zeroes in the minuend (Davis & McKnight, 1980). Furthermore, many U.S. children who do carry out multidigit addition and subtraction correctly can do so only by using a rote procedure. They do not understand crucial aspects of this procedure and cannot relate them to quantities in the English number words or the written number marks (Cauley, 1988; Cobb & Wheatley, 1988; Davis & McKnight, 1980; Ginsburg, 1977; Labinowicz, 1985; Resnick, 1982, 1983; Resnick & Omanson, 1987).

The nature of the place-value and multidigit addition and subtraction errors indicates that typical U.S. school instruction results in many U.S. children building only a *concatenated single-digit* conception of multidigit numbers in which a multidigit number is viewed as several single-digit numbers placed beside each other (Fuson, 1990). For example, the 3 in 5,386 is viewed as being just a 3 (three) and has no connotation as having a value of three hundreds. Likewise, the 1 in 15 is viewed as being just

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[*Child Development*, 1992, **63**, 491–506. 0009-3920/92/6302-0015$01.00]

a 1 (a one) rather than being seen as one ten. Therefore, for many U.S. children doing multidigit addition, the 1 traded over to (written above) the next column to the left is frequently not given either the multiunit value for the new column in which it is written (e.g., in 284 + 183, a value as a hundred coming from 8 tens + 8 tens = 1 hundred 6 tens) or a value as a ten coming from the 2-digit sum in that column (e.g., as a ten coming from the 16 in the sum 8 + 8). Between half and all U.S. third-grade children interviewed in various studies identify the 1 written above the tens and hundreds column as a "one" and not as a ten or as a hundred (Labinowicz, 1985; Resnick, 1983; Resnick & Omanson, 1987), and only 24% of the second and third graders who subtracted correctly identified their trade from the hundreds place as a hundred rather than as a one (Cauley, 1988).

Although Korean children considerably outperform U.S. children on multidigit addition and subtraction problems requiring trading (regrouping, carrying/borrowing), relatively little is known about the solution procedures they use on these problems or about their understanding of place-value and multidigit addition and subtraction procedures. It is not clear whether their computational facility is based on—or at least associated with–understanding of the values underlying trading, or whether the many accurate calculators look more like U.S. children and carry out the procedures only in a rote way. There is a linguistic reason to believe that Korean children might have better knowledge of place value than do U.S. children. Unlike the English number words, which are quite irregular for 2-digit numbers and do not clearly convey the composition of such numbers as tens and ones, the formal[1] Korean number words used in school for calculations are the regular named-value Chinese words in which this composition is evident. The Korean number words have the structure ". . . , eight, nine, ten, ten one, ten two, ten three, . . . , ten nine, two ten, two ten one, . . . , two ten nine, three ten, . . . , nine ten nine, hundred, . . . , three hundred five ten six." Each written digit is said, and then the value of that digit is named: 2,222 is said as "two thousand two hundred two ten two" (in Korean it is *"ee chun ee bak ee ship ee"*). The English system of number words is a regular named-value system for the third and fourth values (each hundred and thousand is named in this regular way), but it is not regular for the second place, naming neither the ten in the teen words nor the ten in the decade words. Miura, Kim, Chang, and Okamoto (1988) found that Korean first graders used the tens and the units in base-ten blocks to make multiunit presentations of five numbers between 11 and 42 (e.g., they used four long tens blocks and two single blocks to show 42) considerably more than did U.S. first graders. The latter instead made unitary sequence/count/cardinal presentations of single units of 42 little ones cubes. Even Korean kindergarten children used multiunit tens blocks more than did U.S. first graders (Miura et al., 1988). The importance of the regular number words for how children think about two-digit numbers is also shown by other studies investigating children in other language groups with regular named tens: Chinese and Japanese children (in the Miura et al., 1988, study), Japanese first graders before any work on tens compared to U.S. first graders after instruction on tens (Miura & Okamoto, 1989), and Japanese-speaking first graders living in the San Francisco area compared to English-speaking first graders living in the same area (Miura, 1987).

Whether Korean children relate this knowledge of tens and ones to 2-digit addition and subtraction with trading is not clear. Nor is it clear how well Korean children understand addition and subtraction of 3-digit numbers. Trades between the tens and the hundreds place can be thought of in two different multiunit ways. One can generalize the reasoning used in the ones place and think about a two-digit sum in any given column as being a ten and some ones (e.g., think of eight plus eight equals sixteen as being a ten and six ones regardless of whether this sum is in the tens, hundreds, or millions column), trade the ten over to the column to the left because that column is ten times larger, and write the six in the given column. This way of thinking ignores the

[1] Two systems of number words are used with children in Korea. One is the native Korean system, which is used informally with preschoolers to count objects in the world. The other is a formal system based on the Chinese system; it is used in school and in all calculations. The native informal Korean system is a regular named-value system just like the formal Korean system (e.g., 12 is "ten two"), except that the decade words (the words for 20, 30, 40, . . .) are not two ten, three ten, four ten, etc. as in the formal system but are special names with only a distant phonetic relation to the words two, three, four.

value of the column as tens, hundreds, millions, etc. and might be used particularly by Korean children because they say any two-digit sum as ten and some ones (e.g., 16 is said as "ten six"). One can also consider the value of the column as tens, hundreds, or millions and think, for example, "eight hundred plus eight hundred is one thousand six hundred so I write six hundred in the hundreds column and write the thousand over there in the thousands column." The former method requires a *regular one/ten trades* conceptual structure: the understanding that the successive values in the words (or successive positions in written multidigit marks) are related to each other by one/ten trades such that ten of a given multiunit (position) are equivalent to one of the next larger multiunit (next position to the left). The second method involves a *multiunit quantities* conceptual structure: the understanding that a multidigit number is composed of a certain number of various sized multiunits. Here the value words in a named-value description of a multidigit number are not just names but are quantities: "four thousand eight hundred five ten three" is understood to consist of four of the special thousand multiunits, eight of the special hundred multiunits, five of the special ten multiunits, and three single units. This conceptual structure might be facilitated in Korean children because every value is clearly named, thus making it easier for Korean children to relate these named values to their column positions. Both of these kinds of conceptual structures can fruitfully direct children's thinking about multidigit addition and subtraction; they are discussed more fully in Fuson (1990).

The purpose of the study reported here was to explore Korean second and third graders' understanding of place value and multidigit addition and subtraction as reflected by their solution procedures and errors on multidigit problems and by the conceptual structures used to explain the trading involved in such problems. Second- and third-grade children in Seoul, Korea, were asked to solve 2-digit and 3-digit problems given in vertical form and then to explain the correctness of already-solved problems. The interviews were given in the middle of the year, when the second graders had studied addition and subtraction of 2-digit but not of 3-digit numbers to see how many of them could generalize their 2-digit procedures and understanding to 3-digit numbers.

Understanding Korean children's multidigit addition and subtraction procedures requires understanding how they carry out the single-digit addition and subtraction within each column. For sums over ten, one addition and two subtraction procedures appear in the school textbooks. All three use the value of ten said in the Korean number words. The *up-over-ten method* for addition[2] splits one addend into (*a*) the number that will make ten with the other addend and (*b*) the left-over number; the answer is then "ten the-left-over-number" as in "eight plus six equals eight plus two (to make ten) plus four (the left-over rest of the six) = ten four." In subtraction, the *down-over-ten method* is the reverse of the up-over-ten method: the subtrahend (the number being subtracted) is split into the number that exceeds ten and the rest which is then subtracted from ten to give the answer. So "ten four − six" (14 − 6) is just the reverse of the example above: "ten four − four (to get down to ten) − two (the rest of the six) = ten − two = eight." In the *take-from-ten method,* the whole subtrahend is taken from the ten, and this difference in added to the number that exceeded ten as in "ten three − six (13 − 6) = (ten − six) and three = four and three = seven." First graders interviewed in the same schools as the second and third graders in this study primarily used these three methods to find sums and differences between 10 and 18. Some children also used more primitive counting methods (Fuson & Kwon, in press).

Method

Subjects.—The subjects were 72 middle-class second- and third-grade children attending two schools in Seoul, Korea. Children must be between 6-0 and 7-0 to begin first grade on March 1 (the first day of school), so at the time of the interview, near the end of the first semester, the second graders ranged in age from 7-4 to 8-4, and the third graders ranged from 8-4 to 9-4. Eighteen children were randomly selected from a randomly selected second-grade and a randomly selected third-grade classroom in each school. There were 19 male and 17 female third graders and 25 male and 11 female second graders, with similar proportions by gender coming from each school (9 and 9, 10 and 8 third graders and 12 and 6,

[2] The names of these addition and subtraction methods are our own names.

13 and 5 second graders). There were about 50 children in each classroom in these schools; this is within the range of 40 to 55 that is presently typical for Korean elementary schools. The teachers of the second graders had completed only the text for the first half of second grade; all 3-digit work is in the text for the second half of that year.

Tasks and procedure.—Each child was interviewed individually in a room in the school. The child was asked to solve two 2-digit addition problems requiring a trade from the ones (27 + 57 and 54 + 19) and two 3-digit addition problems requiring a trade from the tens (284 + 681 and 571 + 293) and then to solve four subtraction problems that were the inverses of the addition problems (and so required the inverse trades from the tens and from the hundreds place). The problems were written in vertical form, and the addition and subtraction problems were on separate pages.

Children were then sequentially shown three addition problems and three subtraction problems that were already solved (see Fig 1). Each problem was written on an index card, and the solution (the answer and any trades) was written in a different color than the problem. Problems *b*, *c*, and *e* were solved correctly using the algorithms shown in the Korean text. Problem *a* contained the common U.S. "vanishing the 1" error (Fuson & Briars, 1990) in which the 1 from the 2-digit sum (e.g., the 1 from the 14 sum of 8 and 6) is ignored (it vanishes). Problem *d* contained the most common U.S. subtraction error (e.g., VanLehn, 1986) in which the smaller number in each column is subtracted from the larger number regardless of their locations in the top or bottom number. Problem *f* contained the common U.S. error in which the first nonzero value to the left of zeroes (here a hundred) is traded for ten ones, and the bottom number under the middle zero is just brought down (or this can be viewed as a subtract smaller from larger error in the middle column: 0 − 6 = 6). The order of problems *a* and *b* and of *d* and *e* was counterbalanced across children within each classroom.

The interviewer told the children that she was going to show them some problems that another child had already solved and that they were to tell whether the child had solved the problem correctly or not. When children did not spontaneously explain why a given problem solution was correct or incorrect, the interviewer asked why they had said it was correct or incorrect. If during the explanation a child did not spontaneously give a value for a traded 1, the experimenter

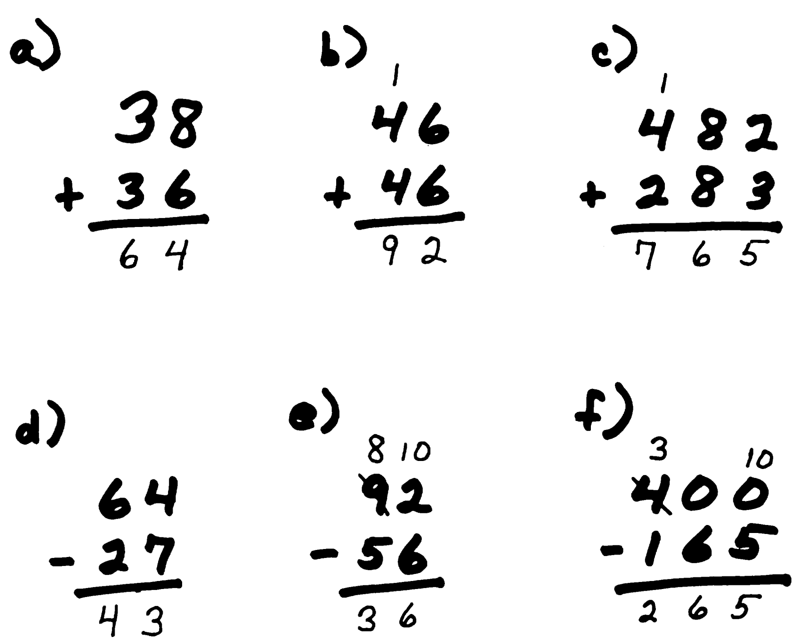

FIG. 1.—Interview problems described as already solved by a fictitious child (the fictitious child's solution is shown by the light lines and the problem is shown by the heavy lines; these were two different colors in the interviews).

then asked an open question like that used with U.S. children by Labinowicz (1985)—"How much is this worth?"—while pointing to the traded 1. If the child did not respond to that probe, a multiple-choice question ("Is this one, ten, hundred, or thousand?") was asked. Interviews were carried out in Korean by the second author, a native Korean experienced in interviewing children. The verbatim responses of the children were written in Korean on interview forms.

Coding of the interview data.—The correctness of each of the four addition and four subtraction multidigit problems solved by each child was evaluated by both authors, and the solutions were classified by the conceptual structure used in that solution. There was complete agreement concerning correctness and the conceptual structure indicated by the child's solution procedure. The verbatim interview data concerning the children's explanations of the six already worked addition and subtraction problems were translated into English by a native Korean speaker different from the interviewer. The first author coded the English translations, and the second author coded the original Korean words. Interviews were coded with respect to whether the child (*a*) demonstrated knowledge of the place-value names "ten" and "hundred" by spontaneously using "ten"' to refer to the second column and "hundred" to refer to the third column; (*b*) demonstrated knowledge of the value given for the 1 traded into and from the tens column and the hundreds column; (*c*) described the quantities involved in the trading procedure; (*d*) showed particular conceptual structures in discussing the problems; and (*e*) demonstrated errors or correct explanations for the complex trading in the subtraction problem with two top zeroes. Particular coding will be described in the Results sections as the results are presented. Intercoding agreement was equal to or greater than 90% in all cases.

Results

Effects of gender and of school.—Because the second-grade sample had so many more boys than girls (25 vs. 11), all of the results were examined for effects of gender. For some tasks, even the second graders were virtually at ceiling, so our data do not permit a determination of gender effects in these areas. In areas that were not at ceiling, there were no effects of gender. Therefore, all data reported below are pooled by gender. Effects of the two different schools were also examined. In several cases differences did appear; in those cases results are reported separately by school.

Solving addition problems with trades.—The problems solved by the children were evaluated for the correctness of the answer and for the solution procedure used to solve each problem. Performance was excellent in both grades: the second graders solved 94% of the addition problems correctly, and the third graders solved 98% of these problems correctly. One second grader made a trading error (vanished the one) on every problem, one third grader made a fact error on both 3-digit problems, and four second graders and one third grader made a fact error on one problem. Thus, even though the second graders had not yet received instruction in school on 3-digit addition problems, their performance on such problems was indistinguishable from that on the instructed 2-digit problems and was virtually indistinguishable from that of third graders who had received such instruction both in the second half of second grade and in the first half of third grade.

Second graders solved all addition problems with the algorithm usually used in the United States: the traded one ten was written as a 1 above the tens column, and the traded one hundred was written as a 1 above the hundreds column, as in problems *b* and *c* in Figure 1. For third graders, 58% of the problems were solved with the U.S. algorithm, 37% were solved by an abbreviation of this algorithm in which the trade is added mentally into the requisite column and is not written above that column, 4% (all by one child) were solved by a variation of the first procedure in which the traded 1 was written beside the smaller of the two numbers in the traded-to column, and 4% (all by one child) were solved by a variation of the first procedure in which the traded 1 was written above the traded-to column but the sum of that traded 1 and the top number was written beside and between these two numbers. These last two variations make it easier to find the sum of the three digits in a traded-to column.

Solving subtraction problems with trades.—For problems on which they had received instruction in school, these children were very accurate and had virtually mastered trading, the nemesis of U.S. children. Second graders were accurate on 94% of the 2-digit problems. Every second grader traded correctly on every 2-digit problem;

the four errors were fact errors made by different children. Third graders were accurate on 100% of the 2-digit problems and on 93% of the 3-digit problems. Four of the five errors on 3-digit problems were ambiguous because nothing was written in the problem except the answer; they all could have reflected incomplete trading or a fact error because the answer in the traded from column was one too large.

Second graders were accurate on 78% of the 3-digit problems, which they had not yet worked in school. The errors made are classified by conceptual structure in Table 1. Thirty of the 36 children successfully traded a hundred on both 3-digit problems; four of these children made an error doing a single-digit subtraction on one of these problems. The remaining six second graders did not extend their correct 2-digit ones/tens trading procedure to a tens/hundreds trade: they made some kind of trading error on both 3-digit problems. However, not a single Korean second grader made the most common U.S. error of subtracting the smaller number from the larger number when the smaller number is on the top, and only one child made an error reflecting a concatenated single-digit conceptual structure. This was a compensation error in which just enough "ones" are taken from the column to the left to make the top number equal to the bottom number; U.S. children also make this error. Three Korean second graders made an interesting variation of this error that reflected a "regular one/ten trades" conceptual structure (see Table 1): two or three tens rather than two or three single units were traded from the left. One second grader did an incomplete trade on both problems, writing a 10 above the tens column but not reducing the hundreds column by one. The sixth second grader demonstrated a "multiunit quantities" structure by trading a hundred into the tens place, but evidently could not coordinate that hundred with the 6 tens already there (see Table 1). Thus, five of the six second graders unable to devise a successful hundred/ten trading strategy nevertheless demonstrated use of multiunit quantities (hundreds, tens, or ones) rather than a rote procedural or concatenated single-digit approach to the problem. All six of these second graders also were systematic in their approaches to the 3-digit problems, with each child making exactly the same kind of error on both such problems.

Over both grades, children used nine different correct solution procedures (see Table 2). Children were quite consistent in the solution procedures: 30 third graders and 22 second graders used only a single solution procedure across all four problems, 5 third graders and 13 second graders used one solution procedure on both 2-digit problems and a different solution procedure on both 3-digit problems, and one child at each grade used one solution procedure on three problems and a different procedure for the fourth problem. On 2-digit problems, all but one second grader and about a third of the third graders used the solution procedure

TABLE 1

ERRORS BY SECOND GRADERS ON 3-DIGIT SUBTRACTION PROBLEMS

	Compensation to Make Top Number Equal to (or Just Larger Than) the Bottom Number			Partially Correct Trading Procedure
Concatenated single-digit conceptual structure	7 2 ~~9~~ 6 5 −2 8 4 5 0 1			
Regular one/ten trades conceptual structure	7 20 ~~9~~ 6 5 −2 8 4 5 0 1	20 9 6 5 −2 8 4 6 0 1	6 30 ~~9~~ 6 5 −2 8 4 4 1 1	10 9 ~~6~~ 5 − 2 8 4 7 8 1
Multiunit quantities conceptual structure				8 100 ~~9~~ 6 5 − 2 8 4 8 0 1

TABLE 2

NUMBER OF CHILDREN USING PARTICULAR SUBTRACTION SOLUTION PROCEDURES BY GRADE, CONCEPTUAL STRUCTURE, AND TRADE POSITION

	CONCEPTUAL STRUCTURE UNDERLYING THE SOLUTION PROCEDURE			
	Concatenated Single Digit	Multiunit Quantities	Regular One/Ten Trades	No or Partial Written Trades
		Solution Procedures for 2-Digit Problems		
Solution Grade 2, 3		8 10 ~~9~~ 6 (*N*'s = 35, 14)		9 6 (*N*'s = 1, 16)
Solution Grade 2, 3		8 10 +2 ~~9~~ 6 (*N*'s = 0, 1)		8 ~~9~~ 6 (*N*'s = 0, 2)
Solution Grade 2, 3		8 10 +16 ~~9~~ 6 (*N*'s = 0, 1)		
Solution Grade 2, 3		8 16 ~~9~~ ~~6~~ (*N*'s = 0, 2)		
		Solution Procedures for 3-Digit Problems		
Solution Grade 2, 3	8 1 ~~9~~ 6 5 (*N*'s = 3, 2)	800 100 ~~9~~ 6 5 (*N*'s = 1, 1)	8 10 ~~9~~ 6 5 (*N*'s = 22, 9)	9 6 5 (*N*'s = 2, 16)
Solution Grade, 2, 3		8 100 ~~9~~ 6 5 (*N*'s = 3, 1)	8 10 +2 ~~9~~ 6 5 (*N*'s = 0, 1)	8 ~~9~~ 6 5 (*N*'s = 0, 2)
Solution Grade 2, 3			8 10 +16 ~~9~~ 6 5 (*N*'s = 0, 1)	
Solution Grade, 2, 3			8 16 ~~9~~ ~~6~~ 5 (*N*'s = 0, 4)	

portrayed in Korean textbooks for 2-digit problems: a ten is taken from the tens column (shown by crossing out the tens digit and writing the number one less than that digit), and this ten is written above the ones column as 10 (see the top example in the multiunit quantities column). This Korean form of writing the trade facilitates use of both Korean ten-based methods of finding single-digit differences. Half of the third graders did not write any extra marks or only crossed out the tens digit, evidently able to handle trading procedures mentally and not needing the support of the written 10 to carry out the single-digit subtraction. Four third graders did variants of the standard Korean procedure in which they wrote the difference between the number being subtracted and ten (this facilitated the single-digit subtraction), wrote the sum of the traded ten and the top number, or wrote the ones as a 2-digit number (these are the second, third, and fourth entries in the multiunit quantities column in Table 2).

The Korean textbook shows the same procedure for 3-digit numbers as for 2-digit numbers: a 10 is written above the tens column to show the new traded tens from the hundreds column. Because a 10 rather than the actual borrowed 100 is written, this procedure for 3-digit numbers reflects a "regular one/ten trades" conceptual structure rather than a "multiunit quantities" conceptual structure. Within a "regular one/ten trades" conception of multidigit numbers, the trading and the single-digit subtraction is identical in every column; for example, 16 − 8 is thought of as "ten six minus eight" whether it occurs in the ones column or the tens column. Even though they had not yet seen this standard Korean procedure done in school for hundreds/tens trades, 60% of the second graders used it correctly on both of the 3-digit problems. A fourth of the third graders used this procedure on 3-digit problems, and as in addition, half of the third graders showed no trading marks at all or only crossed out the hundred. The three variants of the standard procedure in which extra numbers were written reflect a regular one/ten trades structure when used for a hundred/tens trade because a ten rather than a hundred is written for the traded hundred; six third graders used such variants. Six children used solution procedures that showed the traded hundred as 100 rather than as 10; four of these were second graders. Five other children showed the trade by writing a 1 rather than a 10 or 100 above the top number. Overall, 17 children used some solution procedure other than the standard procedure or an abbreviation of it, 31 used the standard procedure, and 20 used an abbreviation of the standard procedure.

Identifying the tens and hundreds places.—During the discussion of the correctness or wrongness of the six already solved problems presented in the interview, all of the second and third graders spontaneously identified the second position as the "ten" place[3] or called a digit written in that position "ten" (e.g., called a 5 in 52 "five ten"), and all of the third graders spontaneously identified the third position as "hundred." Of the second graders, 31 of the 36 spontaneously identified the third position as "hundred." Two of the remaining five used the word "hundred" in saying a 3-digit answer to a problem; the other three never said "hundred" but were never asked to say a 3-digit number or the name of the third position, so their knowledge is not clear. These spontaneous responses were not just answers to questions about the name of given positions; they were all uses of these names in discussing multidigit addition and subtraction. Thus, these Korean second and third graders demonstrated considerably more knowledge of the names of the second and third positions than do U.S. third graders, who frequently cannot even label either position.

Explanations of the addition trading procedure.—In the interview, all of the children said that the correctly solved 2-digit addition problem was correct, and all but one child at each grade said that the correctly solved 3-digit problem was correct. All of the third graders and 34 of the 36 second graders said that the incorrectly solved 2-digit addition problem was incorrect.

The number of children giving specified values for the 1 mark written at the top of the tens and the hundreds column is given in Table 3. In sharp contrast to the frequent identification of the value of this mark as "one" by U.S. children, not a single Korean child so identified the 1 in the tens column, and only one second grader did so for a 1 written in the hundreds column. Children instead spontaneously or when asked the open question identified the value of the 1 written in the tens column as a ten; about two-thirds did so spontaneously as part of their justification of the correctly solved problem or criticism of the incorrectly solved problems. Five children did not spontaneously say anything about the 1 and were, through experimenter error, not asked the questions about the value of the 1 in the tens column.[4] Eighteen second graders and three third graders identified the 1 written in the hundreds column as a ten rather than as a hundred, but almost all third graders identified this 1 as a hundred. Thus, integrating the generalization of the trade of the ten from a 2-digit single-digit sum with the named-value meanings of the places may be something that occurs with instruction or with practice as children solve such 3-digit problems in school. The second graders from School A demonstrated more awareness of the ten and hundred values than did the second graders from School B: significantly more School A children spontaneously identified the ten and/or hundred val-

[3] The Korean language does not distinguish singular and plural forms; these are inferred from the context. Thus, one cannot say "four tens" but only "four ten." Our discussions sometimes use singular and plural form to make the English reading smoother, but our translations of Korean children's descriptions use only the singular English form. The reader should be aware that the child might well have a plural meaning for our translated singular form.

[4] Three of these were asked about the 1 in the hundreds column and said that the 1 was a hundred (so presumably also knew the value of the 1 in the tens column as a ten), one was the second grader who said the 1 in the hundreds column was a one (and thus might have said the same about the 1 in the tens column), and one was a third grader not asked the value of the 1 in either column.

TABLE 3

NUMBER OF CHILDREN GIVING SPECIFIED VALUES TO THE TRADED 1 IN ADDITION PROBLEMS

	ADDITION	
COLUMN IN WHICH THE 1 IS WRITTEN	Second Grade	Third Grade
Tens column:		
Spontaneous identification as a:		
one	0	0
ten	22	22
Identification to the open question[a] as a:		
one	0	0
ten	12	11
No probe question was given	2	3
Hundreds column:		
Spontaneous identification as a:		
one	0	0
ten	0	1
hundred	8	12
Identification to the open question[a] as a:		
one	1	0
ten	18	2
hundred	8	20
No question was given	1	1

[a] This question was "How much is this worth?" said while pointing to the 1.

ues of the 1, 15 versus 8, $\chi^2(1, N = 36) = 5.90$, $p < .05$, and significantly more children identified the hundred as a hundred either spontaneously or in response to the open question, 12 versus 4, $\chi^2(1, N = 36) = 7.20$, $p < .01$.

Korean children used a range of terms to describe the trading. The traditional Korean phrase for describing this trading is to use a form of *pada-olleeda* from *pada* (take, get) and *olleeda* (raise); *pada olleeda* thus has the connotation of *taking* the value from a digit and *raising* it up to the next higher value. Children, however, often used the abbreviated form *olleeda*. This has the same meaning as does "raise" in English, and children's use of *olleeda* might be translated as "I raise it." The Korean term *olla-gada* is the active voice form of the passive form *olleeda* and might be translated as "it goes up." Children used each of these forms with "it" ("it goes up" and "I raise it"), "one" ("one goes up" and "I raise one"), and "ten" ("ten go[es] up" and "I raise ten"). A given child frequently used two or more of these different terms across different problems or even on the same problem. The fact that every child who used "it" or "one" was asked the value of the 1 and responded that it was ten indicates that children knew quite well that the referent for "it" or one is a ten. For both terms, the words themselves imply that when the 1 is written in the next left column, it is not just written there but has its value raised by being in that column.

Children's use of the "go up" and "raise" phrases were usually preceded by a justification. This consisted either of a general rule ("when the sum is more than ten") or, more frequently, of using the specific 2-digit or 3-digit sum in the problem, as in "Because six plus six is ten two, one goes up." Occasionally children gave fuller justifications such as "the ten from the ten two goes up," but the connection was usually not specified, perhaps because the form of the sum (always containing the word "ten") makes what goes up (a ten) so obvious to the child. The inherent meaningfulness of the Korean terms for trading in fact was the reason the question about the value of the 1 was occasionally omitted: to the Korean experimenter, children using the traditional phrase seemed to understand the value shift, so the follow-up question to verify any ambiguous response was not always used.

Children's discussions of the interview

addition problems were classified as to the underlying conceptual structure used to describe the multidigit numbers. No child used a concatenated single-digit conceptual structure because every child identified the 1 traded into the tens column as a ten, not as a one. Children did use a multiunit quantities conceptual structure that named the values of all three digits (ones, tens, hundreds), a regular one/ten trades conceptual structure that viewed each column as single digits that could sum to a ten and some ones, and a combination of the two in which part of the explanation used one structure and part used the other. The difference between the multiunit quantities and the regular one/ten trades conceptual structures is evident only when discussing adding the tens column: a multiunit quantities description is "eight ten and eight ten are one hundred six ten," while a regular one/ten trades description is "eight and eight are ten six." The mixed structure consists of a problem description in which some values are named and others are not. Some children showed considerable flexibility in such mixed descriptions, as though they were aware of the value at all times and it was almost arbitrary whether it was named or not. Examples are:

For 482 + 283: "Two and three make five. Eight and eight make six ten. Really one hundred go up. One hundred and four hundred make five hundred. Five hundred and two hundred make seven hundred."

For 482 + 283: "I add eight to eight. The sum of these is more than one hundred, so one hundred go up."

For 482 + 283: "Eight ten and eight ten make one hundred six, no it is one hundred six ten. Four and two make six. And the number in the hundred column becomes seven hundred by adding one."

The number of problems discussed and the number of children using each conceptual structure is given in Table 4. Discussion of some problems, especially by the second graders, did not refer to the digits or only referred to the ones digits, so the multiunit quantities and regular one/ten trades conceptual structures could not be differentiated; these problems were omitted from the classification. Across all six addition and subtraction problems, 17 children used all three conceptual structures, 39 children used two, and 16 used only one. There was a difference between the two schools in the structures used to discuss the problems. Significantly more children in School A than in School B used multiunit quantities but not regular one/ten trades descriptions, 16 versus 2, $\chi^2(1, N = 72) = 14.52$, $p < .001$. Significantly more children in School B than in School A used regular one/ten trades and not multiunit quantities structures, 29 versus 11, $\chi^2(1, N = 72) = 18.23$, $p < .001$.

Explanations of the subtraction trading procedure.—In the interview, 71 of the 72 children said that the correctly solved 2-digit problem was correct and that the incorrectly solved problem was incorrect; 19 of the 36 second graders and 33 of the 36 third graders identified at least one error in the incorrect 3-digit problem with two zeroes. Every child identified the trade from the tens to the ones column as a traded ten.

The children used four main terms to describe trading in subtraction: borrow,

TABLE 4

NUMBER OF PROBLEMS AND NUMBER OF CHILDREN SHOWING SPECIFIED CONCEPTUAL STRUCTURES IN DISCUSSING ALREADY-SOLVED PROBLEMS

Conceptual Structure	Second Grade: School A	Second Grade: School B	Third Grade: School A	Third Grade: School B
Addition problems:				
Multiunit quantities	14 (10)	3 (3)	20 (11)	5 (3)
Mixed multiunit quantities and regular one/ten trades	10 (10)	4 (3)	6 (5)	12 (8)
Regular one/ten trades	6 (6)	29 (16)	17 (10)	30 (16)
Subtraction problems:				
Multiunit quantities	18 (11)	5 (4)	16 (11)	14 (10)
Mixed multiunit quantities and regular one/ten trades	6 (5)	11 (9)	13 (8)	6 (6)
Regular one/ten trades	11 (8)	31 (16)	23 (13)	32 (16)

NOTE.—The number of children is in parentheses.

lend, give, and *bada-naelleeda*. The last is the traditional Korean phrase meaning "take down" or "bring down"; it has a value-change meaning similar to the value-change meaning for the addition term *bada-olleeda*. In Korean, "lend" is a compound word including the meaning of "give," so children's use of "give" in this context is probably an abbreviated form of "lend." Second graders in School A were the only heavy users of the traditional term *bada-olleeda*. "Borrow ten" and "lend ten" were the most frequently used phrases.

The conceptual structures children used in discussing the already-solved 2-digit and 3-digit subtraction problems are given in Table 4. These structures are fairly similar to those used for the addition problems except that third graders in School B used the pure multiunit quantities structure somewhat more than in addition. These increases may have stemmed from the Korean subtraction method of writing 10 instead of a 1 as in addition; writing 10 may have reminded children to say the value word when discussing the trading. As with addition, significantly more children in School A than in School B used multiunit quantities and not regular one/ten trades descriptions, 12 versus 3, $\chi^2(1, N = 72) = 6.82$, $p < .01$. More children in School B than in School A used regular one/ten trade structures and did not use multiunit quantities structures, 21 versus 11, $\chi^2(1, N = 72) = 5.63$, $p < .02$.

Data concerning children's discussion of the incorrectly solved interview problem with two zeroes in the top are given in Table 5. This problem was 400 − 165 = 265 with the 4 crossed out to a 3 and 10 written above the ones place (see *f* in Fig. 1). This incorrect solution had two errors: one hundred was traded for ten ones, and the tens column was solved by the common U.S. error 0 − 6 = 6. Most Korean third graders not only corrected the incorrectly solved problem, but also explained the complex multiple trading procedure in a conceptually based fashion and obtained the correct answer mentally without writing anything to help them find the answer. Seven children de-

TABLE 5

CLASSIFICATION OF CHILDREN'S RESPONSES TO THE INTERVIEW PROBLEM 400 − 165 = 265

	SECOND GRADE		THIRD GRADE	
	School A	School B	School A	School B
Trading performance:				
Specified correct trading	3	0	15	16
Got the correct answer	3	0	15	15
Got an incorrect answer	10	18	3	3
Said "I don't know"	5	0	0	0
Trading methods:				
Correct trades:				
Distribute 100 as 9 tens and 10 ones	0	0	1	6
Trade 100 to tens column and 10 of that to ones column	2[a]	0	7	5
Trade 10 to ones columns and have 9 or 9 tens in tens column	0	0	7	5
Incorrect trades:				
Take one hundred and get ten ones	5	12	1	1
No trade needed for the tens column:				
0 − 6 = 6	4	11	1	1
0 − 6 = 0	1	1	0	0
Give a ten to both the ones and the tens columns but do not specify the source of one of these and only reduce the hundreds by one	2	4	1	0
Borrow from the hundreds twice, once for the ten ones and once for the ten tens	0	4	1	1

[a] The third correct child subtracted left to right and so subtracted the ones value from the difference of the hundred and ten places ("two hundred four ten minus five is two hundred three ten five").

scribed a simultaneous distribution of the borrowed hundred to both the tens and the ones column. Some children described this distribution using multiunit quantities ("I borrowed one hundred. And I gave nine ten to the tens column and the remainder, ten, to the ones column"), while others described it using regular one/ten trades ("As four becomes three, the zero in the tens column becomes nine and the zero in the ones column becomes ten"). Twelve third graders traded a hundred to make ten tens and then traded one of those tens to make ten in the ones column, leaving nine ten in the tens column. Twelve other third graders started with the trade result in the ones column (they said there were ten there without saying where they came from) and then when discussing the tens column said that there would be nine (or nine tens) in the tens column. Some of these began by describing borrowing ten from the zero in the tens column, for example, "Five cannot be taken from zero. I borrow ten from zero and subtract five from ten. By taking ten from zero, nine ten minus six ten is three ten. Three hundred minus one hundred is two hundred." The three second graders who solved the problem correctly were all from the school giving more multiunit quantities explanations (School A).

Five second graders from School A said they did not know how to do the problem; two of these showed quantitative knowledge in solving part of the problem but then got stuck. The rest of the second graders and five third graders showed the errors described in Table 5. These errors showed different amounts of understanding. Nineteen children accepted as correct the incorrect trade of one hundred for ten ones shown in the problem. Most of these children also agreed with the incorrectly solved tens column ($0 - 6 = 6$); two children instead asserted that zero minus six was just zero. If children simply talked through the tens column shown in the problem (e.g., said "zero minus six is six"), they were asked, "Can you minus six from zero?" to ascertain whether they really thought so or were just saying so without thinking about it. Two children then moved from the first to the last incorrect trade explanation in Table 5, but most children reasserted that zero minus six was just six (as do many U.S. children). The second and third incorrect trade errors in Table 5 show more knowledge than the first because these children tried to cope with the need to get more in the tens column as well as in the ones column. There was a difference between schools in the second graders' approach to this problem in that more children from School A used multiunit quantities in attacking the problem. Children in this school produced the only three correct solutions, and significantly fewer of them used the incorrect trades that violated the multiunit quantities, 7 versus 18, $\chi^2(1, N = 36) = 8.97, p < .01$.

Discussion

These Korean children showed exceptional competence in multidigit addition and subtraction, and their solutions were based on quantitative understanding of multidigit numbers. Almost all children explained the ones/tens trading for both addition and subtraction as involving the value ten, rather than only the value one as do many U.S. children. The facility shown by third graders on a 3-digit problem with two top zeroes was especially striking: most were able to do the problem mentally while looking at the misleading incorrect solution and were able to articulate reasons for their solution procedure based on the multiunit values involved. Korean children seem to be very aware of the values of the ones, tens, and hundreds places, and use these quantitative values in explaining the single-digit addition/subtraction within values and the one/ten trading between columns.

Korean children have several linguistic and instructional supports that are not usually available to English-speaking children in the United States. These supports actually make the cognitive tasks involved in multidigit addition and subtraction different for children in the two countries. The Korean language explicitly names sums between ten and nineteen as ten and some ones, so in an addition problem single-digit sums over nine in any column are already given in a form that cues trading. The teaching of the over-ten method for addition further emphasizes the ten within these single-digit sums. The Korean child still might have to make a shift from thinking of the ten as a collection of ten ones to thinking of it as one ten (to be written as 1 instead of as 10), but this shift is supported by the traditional Korean phrases used to describe writing the 1 (ten) in the next left column: these phrases refer explicitly to a value increase. In subtraction, all three of these supports are available (though the traditional value decrease words were not used by most children), and the Korean subtraction algorithm—in which 10 is written above the column to the right to show the ten traded there—provides further support for the shift from one ten to ten ones. In contrast, the presentation by English number words of numbers between ten and nineteen as unitary collections of ones results in English-speaking children having an additional task of seeing or making the ten ones in this unitary collection of ones without any verbal cue to do so. Most of the traditional English words used to describe the value shifts required by sums of ten or more provide less support than do the Korean words: "carrying" only tells what to do with a 1, "regrouping" says to do something with entities but is not explicit about what is regrouped, and "trading" does convey some sense of fair exchange but is not explicit about the value of ten involved in the trade.

These Korean supports obviously are very strong for the first trade, that between the ones and the tens column. For trades between other columns, there is a tension between operating in a given column while ignoring its multiunit value (using a regular one/ten trades conception) and thinking about the actual multiunit values while adding and subtracting (using a multiunit quantities conception). Some Korean children explained addition and subtraction using each of these conceptions, and many Korean children showed facility in moving back and forth between these two kinds of conceptions. The standard Korean written procedures are consistent with a regular one/ten trades conception, but multiunit conceptions were reflected in some children's written procedures. The multiunit quantities conception did seem to empower some second graders to figure out the 3-digit problem with two zeroes in the top and to protect them from erroneous trading procedures, and there were differences between schools in the extent to which children used a multiunit quantities conception. Thus, supporting such a conception with second graders might be explored in Korean schools. However, the evidence concerning how Korean children labeled the traded 1 in addition indicates that even though many second graders may consider the 2-digit sum in the tens column to consist of tens and ones, most third graders had integrated this view with a simultaneous consideration of the multiunit value of that column as tens and thus considered the traded 1 to be a hundred. This suggests that, at least under instructional conditions that do not strongly support use of a multiunit quantities conception, children might first construct and use a regular ones/tens conception and only later be able to consider and relate the ones/tens conception and the multiunit quantities conception.

There are also other sources of numbers structured around ten that are available in the Korean culture but not in the U.S. The metric system, with its one, ten, hundred, and thousand values in all areas of measure, is used in Korea, and the ten-based abacus with its easily related subbase of five has been used as a calculating device for centuries. The abacus is not taught in school in the first three grades, so this is not a direct source of ten-structured thinking for children. But all of these sources of tens and multiples of tens must permeate the thinking of adults who teach children at home and at schools, thereby increasing the probability that their explanations and procedures will be structured around ten. Thus, both parent explanations at home and teacher explanations at school might be more focused on the multiunit quantities, at least of tens and ones. The textbooks used in these schools do have pictures of objects that show the relative sizes of hundreds, tens, and ones, and these pictures are carefully linked to the addition and subtraction procedures. An intermediate procedure of writing partial sums by multiunit value (first the ones and then the tens) is used first, and then the shorter procedure is used. In the textbook explanations, the ones digits are written in red, the tens digits and any expanded tens notation in blue, and hundreds digits and parts of the hundreds solution in green. How much teacher explanations emphasize these multiunit aspects of addition and procedure is an interesting question for future research. The textbooks also show the accelerated placement of multidigit addition and subtraction topics shown in mainland China, Japan, the Soviet Union, and Taiwan compared to the United States (Fuson, Stigler, & Bartsch, 1988).

Korean middle-class children also have other supports for learning mathematics. Their families set high standards for school performance because success in the highly competitive educational system is the main path to higher education and good jobs. Families support school learning with help at home, the purchase of extra workbooks, and attendance at special after-school lessons. We did not ask the second graders how they knew how to add and subtract 3-digit problems (we actually did not expect such a high level of performance), so it is not clear how many of them generalized the 2-digit procedure and how many of them had already worked such problems at home or in special classes. This might be pursued in a future study. The regular named-value Korean words do facilitate such a generalization, so this generalization might be fairly easy for Korean children to make regardless of where they first try such problems.

Children in the United States typically have none of the supports Korean children have. Unfortunately, U.S. mathematics textbooks even present obstacles to meaningful learning (Fuson, 1990, in press). In these textbooks, multidigit addition and subtraction are approached as a rule-based manipulation of written single digits. Pictures showing multiunit quantities appear briefly if at all, and they often are arranged on a page in ways that are not linked easily to multidigit addition and subtraction. Multidigit addition and subtraction are delayed and maximally distributed across grades. Children first spend a long time doing problems with no trading, thus maximizing their opportunity to construct a concatenated single-digit conception of multidigit numbers. Children often do not have an opportunity to solve 3- or 4-digit problems until at least third grade, so they cannot use the support of the regular named English hundreds and thousands or see the generality of multidigit addition or subtraction over several places. In contrast to this pessimistic approach, research indicates that U.S. children can construct multiunit conceptions of addition and subtraction with the support of materials that physically present multiunit quantities if addition and subtraction with these materials is very closely linked to addition and subtraction with written multidigit marks. With such a supportive environment, even second graders can add and subtract 4-digit numbers with understanding (Fuson, 1986; Fuson & Briars, 1990; Fuson, Fraivillig, & Burghardt, in press; Resnick & Omanson, 1987).

The Korean sample was of middle-class socioeconomic status and may have demonstrated higher levels of understanding than is typical of Korean low socioeconomic status children. The data concerning the poor performance and weak understanding of U.S. children come from children from a range of backgrounds and thus might include depressed achievement by lower socioeconomic children. But data concerning high socioeconomic samples of U.S. children indicate that the levels of understanding in the two countries are really quite different. Most of the U.S. children reported in Kamii (1985) as identifying the 1 in 16 as a one and not as a ten were middle-class suburban children. Davis and McKnight (1980) interviewed third and fourth graders from several schools with comparatively high-achieving students and found not a single child who did 7002 − 25 correctly; these children's procedures instead reflected concatenated single-digit conceptions of multidigit numbers.

It is difficult for bright native English-speaking adults (e.g., many readers of this article) to appreciate the cognitive difficulties faced by English-speaking children in seeing the ten in English teen quantities because such adults have been making the cognitive shifts for so long and readily know the value of all teen words as a ten and so many ones. Many U.S. first and second graders do not know such values; they count up from ten to a given number to find such values (e.g., Steinberg, 1984). The extent of this difficulty in children, and the need for support in making the cognitive shifts, is exemplified by an intermediate step invented by high socioeconomic and high-achieving first graders who used multiunit materials for multidigit addition (Fuson, 1986). When these children began to do problems without the materials, they understood that they needed to put the ten from a two-digit sum in the column to the left but in adding they stated these sums to themselves in English teen words (e.g., "six plus six is twelve" or "eight plus five is eight, nine, ten, eleven, twelve, thirteen"). They did not immediately know the composition of these teen words as a ten and some ones, but they had learned to understand written place-value notation as tens and ones. So they used a rote association between the English word and its written place-value numeral ("twelve" is written 12) to write this numeral (12) out at the side of the problem. They then looked at the 12 to see that it was one ten and two ones. Most of these children, and the second graders who were shown this step and used it in subsequent studies, eventually did not need the support of the written numerals to find the ten and the ones in an English word between ten and eighteen, but they did need this support initially. Korean children do not have to find the ten and the ones for a given 2-digit number; they are already given in the Korean number words. This example underscores the special difficulties imposed on English-speaking children and suggests that it might be helpful for them to use "tens words"—English versions of Korean words—to support their multiunit thinking.

These results indicate that what appears on the surface to be the same cognitive or instructional task may actually be substantially different tasks for the children in two different cultures. Spoken language, written symbols, specifically taught solution strategies, and various other aspects of a culture can support or interfere with children's construction of conceptual structures that facilitate learning of the task. Comparative analyses of the conceptual structures used in a task across different cultures can disembed that task from cultural characteristics that might otherwise seem to be inherent in the task, identify conceptual structures that facilitate learning the task, and suggest cultural or linguistic supports that might be helpful in the disadvantaged culture.

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