

***Math Expressions* Interactive Digital Learning Environments For Covid Remote Teaching 2020/2021**

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First, we describe the interactive digital learning environments for each grade, then we give information about using these environments (moving things around, etc.), and finally we give information about how to use these within your district webinar interface. You can access all of the *Math Expressions* Interactive Digital Learning Environments [here](#).

For additional materials, videos, and support using these materials, please visit:

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Description of the Interactive Digital Learning Environments

Math Expressions has carefully designed classroom environments for learning specific concepts. In kindergarten and early first grade these involve manipulatives that children move to make numbers or relationships. There are some manipulatives for every grade. Children describe what they see, compare and contrast what they and others make, and learn vocabulary and concepts as they interact with this manipulative learning environment. Children also make math drawings at all grade levels, often using the special large MathBoards that contain specific grade-level learning supports. If schools have to do some or all teaching remotely during the 2020/21 school year, we need to have digital versions of these learning environments in which children can move and make things and draw. As in the classroom, the teacher needs to see what all children are making so that feedback is possible, and the teacher can select examples to discuss with all children. Supporting teachers as much as possible to orchestrate a Nurturing Math Talk Community in the classroom is vital for student and teacher emotional and conceptual functioning. The need for interactive digital manipulatives is increased by concerns about children sharing any materials in the classroom including those manipulatives used for the Daily Routines and Quick Practices.

Manipulatives: The crucial learning environments we have built are listed below by grade level. Most of them are in kindergarten, but there are some in other grades. Some of these manipulatives are in iTools on the HMH Think Central website, but iTools cannot be saved or seen by the teacher all at the same time because only one child at a time can share their screen. Nor can more than one iTool be on the screen at the same time, and relating different tools is important. We built Google Drive slide decks of the needed manipulatives. Teachers only need the free access to Google to use these slides. When teaching remotely, teachers will give links to each student so that each student is working on their own slide. The teacher can see what a student is doing on each slide and can guide the class as a whole or individual students as in the

classroom. Students can also see the teacher’s manipulatives so that the teacher can orchestrate the activity. Logistics about doing this is in Part C of this document.

MathBoards: Unfortunately, Google does not have a good pen for writing on the slides. This is not necessary with the manipulatives, but math drawings are central to the *Math Expressions* classroom. So, we have made the *Math Expressions* MathBoards for each grade in Jamboard (a free Google app). Students can write on these as they do with the physical MathBoards, and the teacher can see the MathBoards of all students. These provide a general writing surface for math drawings for any occasion, and students or the teacher can save what has been written. The pen currently does not work on the iTools MathBoards, though it does on the Class Presentation MathBoards. But these Class Presentation MathBoards do not save the work consistently, and it is not clear whether users of pre-2018 editions of Math Expressions will have access to the Class Presentations. The physical *Math Expressions* MathBoards are better than digital ones. But with schools contemplating alternating days of live and remote learning for parts of a class, students would need a home and an in classroom MathBoard, which many districts could not do. If your district is completely remote learning, we suggest sending home the MathBoards and students can bring them back to school when school reopens. The digital MathBoards have some advantages and visual supports that the physical MathBoards do not have, so it is good to have both. We also built manipulative environments on the Jamboard MathBoards such as putting in Grades 1, 2, 3, and 4 Secret Code Cards on the Number Path side, putting in Grade 4 and 5 the Fraction Poster in a format where each fraction bar can be copied out to the side and used to compare and find equivalent fractions, and in Grade 4 and 5 creating place-value environments that use the Place-Value Poster and other place-value formats with whole number and decimal Secret Code Cards.

Most lessons can be taught interactively remotely by teachers using visual representations from the TE or from the SAB and screen sharing. Students can discuss things and ask questions. They can make math drawings on a MathBoard or in a notebook or on loose paper. They can use the SAB physically or digitally. **But the Jamboard MathBoards described below, the digital manipulative environments, the Daily Routines, and the Quick Practices will increase the quality of the Math Talk and thinking considerably if learning must occur remotely.**

Kindergarten

The Kindergarten Interactive Digital Learning Environments:

Counting Mats (Units 1 through 3):

- 1 to 5: Purple Number Tiles numeral fronts with numerals and backs with dots and blue and red square-inch tiles fronts with the circle (see SAB p. 5); break-apart stick; orange and yellow circles (having 4 different colors will be helpful in the digital environment).
- 1 to 10: Purple Number Tiles fronts and backs, blue and red square inch tiles fronts and backs (with and without circle), break-apart stick; yellow and orange circles.
- 1 to 10: The above with numbers and dot cards out of order; children put both in order.
- 1 to 10: The above with 5-square tiles blue and red (see SAB 63), 5-group yellow and orange circles, and + and – green tiles.
- 1 to 10: The above with = and \neq green tiles.

Making Partners: Purple Number Tiles numeral fronts with numerals and backs with dots and blue and red square-inch tiles fronts with the circle (see SAB p. 5); break-apart stick; orange and yellow circles; + – = ≠ green tiles

Comparing Mat (Units 1 and 2): This is horizontal only because it is difficult to fit vertically on a slide (see SAB p.25), blue and red square inch tiles circle side only, orange and yellow circles, purple Number Tiles 1 to 10

1 to 10 Board (Unit 2): Large board with ten columns and the numbers 1 to 10 above a column; 5-Counter Strips with blue circles and individual blue circles (see SAB 69); 5-square strips red with circles on them, red tiles with circles

2D Shapes (Units 1 through 3): These include all the shapes in the foam special *Math Expressions* 2D shape Set and are organized with questions to guide lessons with the shapes. These lessons can be done together instead of spread across the first three units, as requested by many Kindergarten teachers.

Making teen numbers: 10-Counter Strip blue circles and individual circles (see SAB 123); 5-square strips red with circles on them, red tiles with circles; Purple Number Tiles 1 to 10 fronts with numerals and backs with dots; + – = tiles

11 to 20 Board (Units 4 and 5): Large board with ten columns and the numbers 11 to 20 above a column; ten-centimeter sticks and single centimeter cubes

MathBoard (made in Jamboard so students can write on it): The Number Parade is on page 1, and then come 8 pages of supports for writing numerals for practice, and a blank page at the end for drawing anything.

The **Daily Routines** are absolutely crucial for learning to count to 100 by ones and by tens and to understand the base-ten relationships within the counting words. These routines require multiple posters and number cards with steps repeated over days to show increasing numbers. Ideally Student Leaders lead these routines, but they could be led in a digital environment by the teacher. The children do not need their own versions (though they could have them for practice or activities at home). Children respond interactively with words and fingers as the Student Leader or teacher leads each routine. The initial Daily Routine continues to 50 when children shift from counting by ones to counting by tens and by ones while using a 120 Poster with columns of ten numbers and a Flip Chart with columns of ten circles. We have made a Google slide deck that lets the Student Leader or teacher interact with the *Math Expressions* manipulatives as the class responds with counting and numerical answers to the Daily Routines.

The **Quick Practices** are also absolutely crucial for learning and practicing concepts in each unit. These require the Number Parade and Giant Number Cards 1 to 10, the Number Pattern (Teen) Poster, and Giant Number Cards with the double wide 10 on the 0 of which the ones cards can be placed to show teen numbers. Ideally student leaders lead these routines, but they could be led in a digital environment by the teacher. The children do not need their own versions (though they could have them to use as center activities). Children will respond interactively with words and fingers as the teacher leads through the Quick Practices in a digital remote teaching mode. We have made a Google slide deck to project that lets the Student Leader or teacher interact with the *Math Expressions* manipulatives as the class responds.

The Daily Routines and Quick Practices build community as students respond together with choral responses. Such feelings of community are especially important now if remote learning must occur.

Grade 1

The Grade 1 Interactive Digital Learning Environments:

Stair Steps and Number Cards (Unit 1 most lessons): The ten blue Stair Steps from 1 to 10 fronts with dots on them and backs plain with a number high on the right end (see SAB 5, 6) are used throughout Unit 1. We have made 14 slides to be used to support these lessons. Children can move the Stair Steps, the Break-apart Stick, and Number Cards.

Red Addition Count On Cards (Unit 2 Lesson 8): Two slides show the backs of these cards with the answer and the number and dots to support counting on. The teacher or students cover all of the answers with a blue square and cover the counting on number and dots with a Hint box. Then students solve each problem using the hint as needed. The teacher explains how to use the hint to count on and then leads the class in answering these additions with totals between 6 and 10.

Yellow Unknown Addend Count On Cards and Orange Subtraction Count On Cards (Unit 3 Lessons 4 and 7): Each kind of card has two slides of cards that follow the same approach as for the Red Addition Count On Cards.

Teen Numbers (Unit 4 Lessons 2, 3, 4): Teen Number Digital Learning Environment for Grade 1 students to make teen numbers with a ten and ones and the Secret Code Cards.

Green Addition Make-a-Ten Cards (Unit 4 Lesson 5): Four slides show these cards that follow the same approach as for the Red Addition Count On Cards.

Blue Subtraction Make-a-Ten Cards (Unit 5 Lesson 2): Five slides show these cards that follow the same approach as for the Red Addition Count On Cards.

MathBoard: The Jamboard *Math Expressions* MathBoard has various pages:

Number Path

Number Path with Secret Code Cards 1 to 9 and 10 to 100 on which children draw ten-sticks and ones and encircle or mark parts of the Number Path to 100

the two 10 by 10 cm dot arrays with Secret Code Cards 1 to 9 and 10 to 100

the 10 by 10 square grid

Blank pages at the end for drawing anything

Daily Routines and **Quick Practices** are equally important for Grade 1. We have made a Google slide deck to project for the Daily Routines and the Quick Practices that lets the Student Leader or teacher interact with the *Math Expressions* manipulatives as the class responds.

Grade 1 students use various strategy/fluency cards for addition or subtraction within 10 or 18. Students will need these cards at home as well as in the classroom. When students first use these cards, it is important to help them see how to use the cards to practice counting on or the make-a-ten strategy using the visual representations on the cards. The cards permit study time to be maximized by focusing on problems and strategies students still need to learn. A card goes into the correct/fast pile, the correct/slow pile, or the oops pile if the answer was not correct. Cards in the correct /fast pile do not need to be practiced except occasionally. **These cards are also in the interactive digital learning environment to be used by the whole class or by individual students.**

Grade 2

The Grade 2 Interactive Digital Learning Environments:

Green Addition Make-a-Ten Cards (Unit 4 Lesson 5): Four slides show these cards that follow the same approach as for the Red Addition Count On Cards.

Blue Subtraction Make-a-Ten Cards (Unit 5 Lesson 2): Five slides show these cards that follow the same approach as for the Red Addition Count On Cards.

MathBoard: The Jamboard *Math Expressions* MathBoard has various pages:

Number Path

Number Path with Secret Code Cards 1 to 9 and 10 to 200 on which children draw ten-sticks and ones and encircle or mark parts of the Number Path

the two 10 by 10 cm dot arrays with Secret Code Cards 1 to 9 and 10 to 200

the 10 by 10 square grid

Secret Code Cards 1 to 9, 10 to 90, 100 to 900 and 1000 around the Number Path so

students can make SCC 3-digit numbers and drawings of hundred-boxes, ten-sticks, and ones.

Blank pages at the end for drawing anything

Daily Routines and **Quick Practices** are equally important for Grade 2. We have made a Google slide deck to project for the Daily Routines and the Quick Practices that lets the Student Leader or teacher interact with the *Math Expressions* manipulatives as the class responds.

Grade 2 students use various strategy/fluency cards for addition or subtraction within 10 or 18. Students will need these cards at home as well as in the classroom. When students first use these cards, it is important to help them see how to use the cards to practice counting on or the make-a-ten strategy using the visual representations on the cards. The cards permit study time to be maximized by focusing on problems and strategies students still need to learn. A card goes into the correct/fast pile, the correct/slow pile, or the oops pile if the answer was not correct. Cards in the correct /fast pile do not need to be practiced except occasionally. **These cards are also in the interactive digital learning environment to be used by the whole class or by individual students.**

Grade 3

The Grade 3 Interactive Digital Learning Environments have two practice tools:

A Multiplication Table with a Target to cover all but one specified row and one column and

A Multiplication Table with a covering bar to practice multiplications and divisions.

MathBoard: The Jamboard *Math Expressions* MathBoard has various pages:

Number Path for showing various multiplications and divisions

Multiplication Table and covering bars to practice multiplications and divisions

Number Path with Secret Code Cards 1 to 9, 10 to 90, 100 to 900 and on which children draw hundred-boxes, ten-sticks, and ones and encircle parts of the Number Path

the 25 by 40 dot array back of the Grade 3 MathBoard

Secret Code Cards 1 to 9, 10 to 90, 100 to 900 and 1000 around the Number Path so students can make SCC 3-digit numbers and drawings of hundred-boxes, ten-sticks, and ones.

Blank pages at the end for drawing anything

There are extensive practice materials for learning multiplications and divisions including various strategy/fluency cards for multiplication and division. Children will need these cards and other materials at home. Many of these materials are in both the Student Activity Book and the Homework and Remembering Book, so the Homework and Remembering Book could stay at home and the Student Activity Book materials could be used in the classroom. These materials are vital for reaching fluency in multiplications and divisions.

Quick Practices are equally important for Grade 3. We have made a Google slide deck to project the Quick Practices that lets the Student Leader or teacher interact with the *Math Expressions* manipulatives and lead the Quick Practice as the class responds.

Grade 4

The Grade 4 Interactive Digital Learning Environments:

These environments can be used when students do not need to write anything. Similar versions on which students can write are in the Jamboard *Math Expressions* MathBoard.

A thousands to ones place-value chart and a Reading Millions Frame with Secret Code Cards: These help with showing, understanding, and reading large numbers.

A hundreds to hundredths place value chart with money below: This generates coins and bills as needed for particular values made with Secret Code Cards to show the resulting number.

The Fraction Poster lengths from one whole to tenths: These show how unit fractions get smaller as the number gets larger and other relationships are also visible.

Movable fraction bars from the Fraction Poster: These can be moved to the right and enlarged and so compare it to another you copied out or make equivalent fractions

MathBoard: The Jamboard *Math Expressions* MathBoard has various pages:

Number Path for drawing anything and reviewing multiplications and divisions as needed

The 50 x 100 dot side of the *Math Expressions* MathBoard for drawing multiplication problems and seeing the hundreds, tens, and ones related to a written method

A thousands to ones place-value chart and a Reading Millions Frame with Secret Code Cards for showing, understanding, and reading large numbers

A hundreds to hundredths place value chart with money below that generates coins and bills as needed for particular values with Secret Code Cards to show the resulting number

The Fraction Poster lengths from one whole to tenths that show how unit fractions get smaller as the number gets larger and other relationships are also visible.

Movable fraction bars from the Fraction Poster that can be moved to the right and enlarged and so compare it to another you copied out or make equivalent fractions

Blank pages at the end for drawing anything

Quick Practices are equally important for Grade 4. We have made a Google slide deck to project the Quick Practices that lets the Student Leader or teacher interact with the *Math Expressions* manipulatives and lead the Quick Practice as the class responds.

Grade 5

The Grade 5 Interactive Digital Learning Environments:

These environments can be used when students do not need to write anything. Similar versions on which students can write are in the Jamboard *Math Expressions* MathBoard.

The Fraction Poster lengths from one whole to tenths: These show how unit fractions get smaller as the number gets larger and other relationships are also visible.

Movable fraction bars from the Fraction Poster: These can be moved to the right and enlarged and so compared to another you copied out or make equivalent fractions.

A symmetric sketch of thousands to thousandths places with expanded notation cells:

This version has numbers filled in to emphasize the symmetry around the ones place and an expanded notation equation of the example number.

The same sketch with all blanks: With this students can make their own number and its expanded notation equation.

A hundreds to hundredths place value chart with money below: This generates coins and bills as needed for particular values and Secret Code Cards to show the resulting number.

A thousands to thousandths Place Value Chart with money below: This generates coins and bills as needed for particular values and there are Secret Code Cards to show the resulting number.

A thousands to thousandths Place Value Chart with Secret Code Cards: This is for showing, understanding, and reading decimal fractions related to fractions as $1/10$ or $1/100$ or $1/1000$ and for comparing numbers including decimal fractions.

Six slides that use the hundredths bar in various ways: These enable students to relate decimal notation to its meaning as money and to understand the difficult point that you can add zeros at the end of numbers to the right of the decimal point without changing the value: for example, $0.3 = 0.03 = 0.003$.

MathBoard: The Jamboard *Math Expressions* MathBoard has various pages:

Hundredths Bar for understanding decimal fractions from the *Math Expressions* MathBoard

The fraction bars on the front of the *Math Expressions* MathBoard

The Fraction Poster lengths from one whole to tenths that show how unit fractions get smaller as the number gets larger and other relationships are also visible.

Movable fraction bars from the Fraction Poster that can be moved to the right and enlarged and so compare it to another you copied out or make equivalent fractions

Hundredths Bars for identifying and comparing decimal fractions

Three labelled Hundredths Bars to relate ways to write the same decimal fraction as tenths, hundredths, or thousandths, for example, 0.4 or 0.40 or 0.400 .

blank pages at the end for drawing anything

Quick Practices are equally important for Grade 5. We have made a Google slide deck to project the Quick Practices that lets the Student Leader or teacher interact with the *Math Expressions* manipulatives and lead the Quick Practice as the class responds.

Grade 6

The Grade 6 Interactive Digital Learning Environments

Two blank factor puzzles: These are for students to make up examples for each other to solve.

Movable fraction bars from the Fraction Poster: These can be moved to the right and enlarged and so compared to another you copied out or use to make equivalent fractions.

The Fraction Poster lengths from one whole to tenths: This shows how unit fractions get smaller as the number gets larger and other relationships are also visible.

A thousands to thousandths Place Value Chart with money below: This generates coins and bills as needed for particular values and there are Secret Code Cards to show the resulting number.

A thousands to thousandths Place Value Chart with Secret Code Cards: These can be used for showing, understanding, and reading decimal fractions related to fractions as $1/10$ or $1/100$ or $1/1000$ and for comparing numbers including decimal fractions.

Two slides that use the hundredths bar in various ways: These enable students to relate decimal notation to its meaning as money and to understand the difficult point that you can add zeros at the end of numbers to the right of the decimal point without changing the value: for example, $0.3 = 0.03 = 0.003$.

MathBoard: The Jamboard *Math Expressions* MathBoard has various pages:

A vertical ratio table, a grid for a Factor Puzzle, a horizontal ratio table, and a graphing environment of 50×50 small squares

An environment of 25×40 larger squares for graphing or drawing geometric shapes

The fraction bars on the front of the *Math Expressions* MathBoard

The Fraction Poster lengths from one whole to tenths that show how unit fractions get smaller as the number gets larger and other relationships are also visible.

Movable fraction bars from the Fraction Poster that can be moved to the right and enlarged and so compare it to another you copied out or make equivalent fractions

Hundredths Bars labelled in different ways for identifying and comparing decimal fractions

Quick Practices are also important for Grade 6. We have made a Google slide deck to project the Quick Practices that lets the Student Leader or teacher interact with the *Math Expressions* manipulatives and lead the Quick Practice as the class responds.

Using the *Math Expressions* Interactive Digital Learning Environments

You need to have a free personal edition access to google to access the slides. Info can be found here: <https://www.google.com/slides/about/> If a teacher has access to Google Classroom, it can be used to make assignments using these slides.

You also need to download Jamboard, a free google application. This can be added to your drive as a free personal version. You can read more information here: <https://jamboard.google.com/> All of the *Math Expressions* MathBoards use Jamboard because it has a better writing tool than the google slides.

Slides, once opened, will force you to make a copy. This will allow the copy you open to be your personal use copy. This is automatically saved in your google drive account. You can read more info on your google drive here: <https://www.google.com/drive/>

Some teachers have Google accounts from their district. They will understand all of this. Others only have personal accounts or no experience. We will explain the basics in our introductory video including a step-by-step guide covering opening the decks, making class sets, saving, and sharing with students.

To move the manipulatives:

- Keep the slides in edit mode, not presentation mode.
- Click the manipulative you would like to move. This will highlight the manipulative.
- Once highlighted, use your cursor (or finger on a touch screen) to hover over the manipulative until you see a cross bar cursor. You may need to move your finger or cursor a bit until you see the cursor. This crossbar indicates you can now move the manipulative freely with your finger or by keeping your mouse pressed down.
- To undo a move, including accidental changes, click the undo button in your toolbar. (upper left, arrow pointing left, and see the diagram on the next page).

The more you practice and the more students practice using the tools, the easier it becomes. These slides were field tested with kindergarteners who caught on in one 15-minute session: Click, let go, wait for crossbar (move finger or cursor a bit to make it appear), move the manipulative, let it go.

When you play, you can move things back to the original setting by using the undo button: the left curved arrow at the top left of the strip after the + and arrow. Or you can leave that slide showing your math thinking to share with someone else. Google slides are housed in Google drive, an internet-based cloud platform. This means all changes and edits are automatically saved to your drive, and you can always go back to earlier versions. **See the annotated slide on the next page for these locations.**

Some of our digital manipulative environments combine manipulatives used on separate *Math Expressions* lessons to enable children to make a variety of visual models and reduce the number of needed slides. For example, the counting mats have the red and blue square inches and two colors of circles on them. With the classroom manipulatives, the squares and circles are used in different lessons. The digital versions allow for additional student choice and flexibility.

Use this undo button to move manipulatives back into place or to undo any move or action.

To move a manipulative, click the manipulative. It will highlight it. Hover your mouse or finger over the highlighted manipulative until the cross-bar cursor appears. This indicates you can now move the manipulative. Move to an edge if the cross bar does not appear and keep moving a bit until it does appear.

Click these three dots and drag them upward to reveal any notes for this particular slide to help you navigate or understand the purpose of that slide.

Setting Up and Teaching with the *Math Expressions* Interactive Digital Learning Environments

We have made written directions with shots of the Google docs environment to explain

- creating a master file.
- creating class sets of slides.
- sharing slides with students.

These directions also include the first steps you should take with the Google Slides, how to rename slides for student use, and how to give students access to the slides with specific sharing methods. The link to these directions is on slide 2 of each grade of the *Math Expressions* Digital Learning Environments. You can also find these directions [here](#).

We also made a video showing the first steps when opening your slide deck, and a video explaining making class sets for Google Slides. The link to each of these videos is on slide 2 of each grade of the Google Slide Digital Learning Environments. You can also find these videos [here](#) and [here](#).

We also made a video explaining how to create class sets of slides for Jamboard, as it is a bit different from the process for Google Slides. The link to this video is on slide 3 of each grade of the Google Slide Digital Learning Environments. You can also find this video [here](#).

Consider having explicit parts of lessons to teach your students how to do the following:

- open a Google Slide or Jamboard via a link.
- find the slide that is theirs.
 - For young students, help them find their slide number and their name on the slide to ensure they are working only on their slide. Practice both of these things.
- move the tools.
 - Highlighting the manipulative (release), waiting for the cross bars, moving it (release)
- fix errors or cleaning the slide at the end.
 - Use the undo arrow (upper left of toolbar) to fix any errors such as placement or manipulative size/shape.
 - Use the undo arrow repeatedly to have the manipulatives return to their spots so that the slide looks the way it did at the start.
 - Or, teach students how to resize objects and physically drag them back to their starting places.

Students can learn all of these things, and it is good general knowledge for them and worth their time.