Teaching the Best Computation Methods

Professor Emerita Karen C. Fuson

Northwestern University

karenfuson@mac.com

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Please see my website karenfusonmath.com for 22 hours of audio-visual Teaching Progressions for all CCSS domains and for my papers, classroom videos, presentations, and supports for teaching remotely.

Inquiry Learning Path

in the Math Talk Community

Bridging for teachers and students by coherent learning supports

Learning Path

Phase 3 | Formal math methods, **fluency**

Math Sense-Making
Math Structure

Math Drawing
Math Explaining

Phase 2 | Research-based mathematically desirable and accessible methods, **understanding and growing fluency**

Math Sense-Making
Math Structure

Math Drawing
Math Explaining

Phase 1 | Student-generated methods, **exploring and growing understanding**

Mathematical Practices			
Math Sense-Making	Math Structure	Math Drawings	Math Explaining
Make sense and use of appropriate precision.	**See structure and generalize.**	**Model and use tools.**	**Reason, explain, and question.**
MP1 Make sense of problems and persevere in solving them. **MP6** Attend to precision.	**MP7** Look for and make use of structure. **MP8** Look for and express regularity in repeated reasoning.	**MP4** Model with mathematics. **MP5** Use appropriate tools strategically.	**MP2** Reason abstractly and quantitatively. **MP3** Construct viable arguments and critique the reasoning of others.
Teachers continually assist students to do math sense-making about math structure using math drawings to support math explaining.			

Teachers continually assist students to do **math sense-making** about **math structure** using **math drawings** to support **math explaining**.

Create a Nurturing Sense-Making Math Talk Community

The teacher orchestrates collaborative instructional conversations focused on the mathematical thinking of students, using responsive means of assistance that facilitate learning and teaching by all.

- Engaging and involving
- Managing
- Coaching*

*modeling, cognitive restructuring/clarifying, instructing/explaining, questioning, feedback

The teacher supports the sense-making of all classroom members by using and assisting students to use and relate:

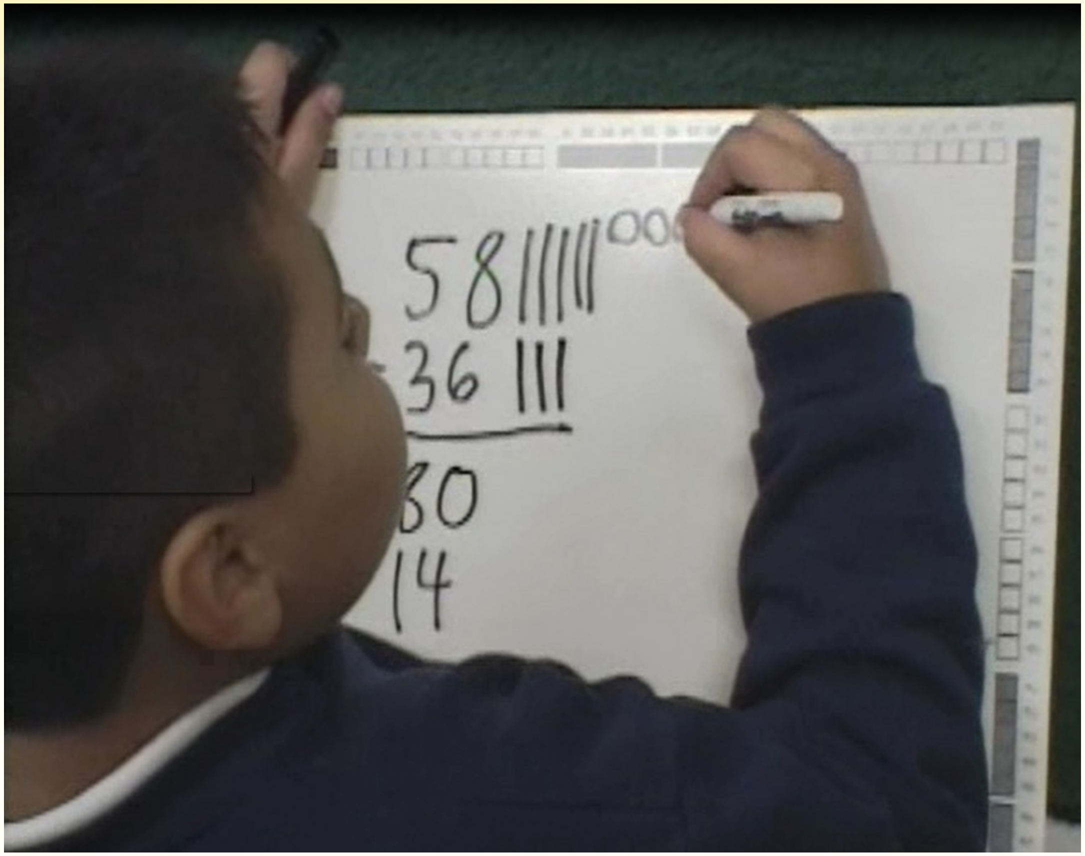
- Coherent mathematical situations
- Pedagogical supports
- Cultural mathematical symbols and labels

Solve and Discuss Classroom Structure

Solve	Explain	Question	Justify
All students solve. Some solve at the board, and the rest at their seats.	**One student at the board** explains and then asks, "Are there any questions?"	**Other students** ask questions to clarify or extend.	The original explainer **responds to the questions** by explaining more (justifying the original explanation).

Any student at any time can ask for help from anyone.
For more practice, Solve and Discuss can take place in pairs or small groups.

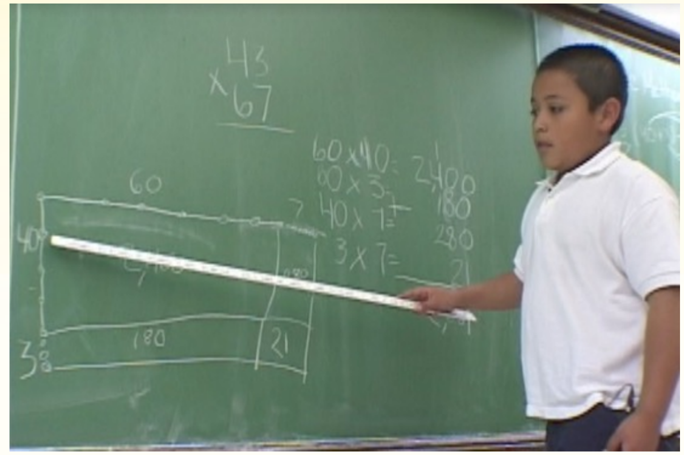
Make the math thinking visible



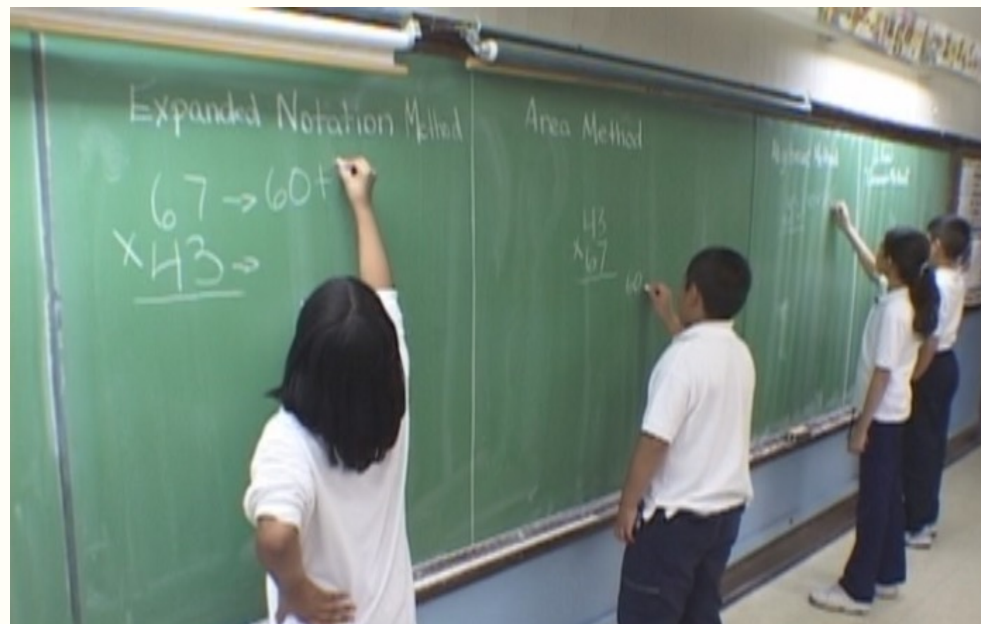
- Students must make some kind of math drawing related to the math symbols to show their thinking.
- This supports understanding by the listeners and promotes meaning.

Make the math thinking visible

- This is important for equity: less advanced students and English Learners are helped by the math drawing linked to the explanation by pointing.



- Be sure that important methods remain on the board or can be made visible again (e.g. on a Math Board) so they can be compared with other methods.



Students must speak and not just listen

1. Structure opportunities to explain to a partner and repeat what the partner says, if needed. Students eventually find their own words, but may need the security of saying an explanation they know is correct.

2. “Bite your tongue” to provide wait time. Students will explain, ask questions, or add a comment if you wait.

3. Help students speak to classmates by moving to the side or back of the room. Later remind students with a silent gesture to address each other.

A nurturing meaning-making visual Math Talk Community:

is an inquiry-based teaching/learning environment, and has continual focus on sense-making by all participants.

Students are expected:

- to understand what they are doing,
- come to be able to explain their thinking,
- understand the thinking of other students,
- learn to seek help when they need it, and
- help others who need it.

Major steps in making computation meaningful:

Relate drawings to numbers to make computations meaningful and do not use drawings just to find answers.

Students later do not use drawings and only use written methods but they can go from numbers to drawings sometimes to retain or recall meanings.

Making computation meaningful:

First, there is no one "standard algorithm."

There are only ways to write computations that people erroneously take to be the standard algorithm.

The first draft of the 2021 California framework is confused about standard algorithms.

There is no one "standard algorithm." There are variations in ways to record efficient, accurate, and generalizable methods that form the collection of standard algorithms.

There are better methods; my research is about these. These are in classroom videos, papers, and Teaching Progressions on my website.

These are the mathematically desirable and accessible methods that are standard algorithms.

Most taken-to-be "standard algorithms" are difficult or misleading.

The CCSS say in the critical area for the first year of a given computation: "Students develop, discuss, and use efficient, accurate, and generalizable methods."

They do not say to wait until Grade 4 to do "standard algorithms."

What Is the Standard Algorithm?

The NBT Progression document summarizes that *the standard algorithm* for an operation implements the following mathematical approach

with minor variations in how the algorithm is written:

•decompose numbers into base-ten units and then carry out single-digit computations with those units using the place values to direct the place value of the resulting number; and

•use the one-to-ten uniformity of the base ten structure of the number system to generalize to large whole numbers and to decimals.

To implement a standard algorithm one uses a systematic written method for recording the steps of the algorithm.

There are variations in these written methods

- **within a country**, **across countries**, and **at different times.**

The standard algorithm is a collection of related ways to write an operation.

What are the best computation methods for multidigit addition, subtraction, multiplication, and division?

I today summarize 20 years of classroom research.

These methods were all created by students. They were then tried in many different classrooms to see if they are accessible. They are. Students, teachers, and parents understand them.

They are more mathematically desirable than other methods.

They are all standard algorithms.

On my website karenfusonmath.com are publications describing the research and these methods:

The Best Multidigit Computation Methods: A Cross-cultural Cognitive, Empirical, and Mathematical Analysis, Karen C. Fuson. Universal Journal of Educational Research 8(4): 1299-1314, 2020 DOI: 10.13189/ujer.2020.080421

Fuson, K. C. & Beckmann, S. (Fall/Winter, 2012-2013). Standard algorithms in the Common Core State Standards. *National Council of Supervisors of Mathematics Journal of Mathematics Education Leadership, 14 (2)*, 14-30. Also at http://www.mathedleadership.org/docs/resources/journals/NCSMJournal_ST_Algorithms_Fuson_Beckmann.pdf

Watch on my website videos from public school classrooms with children from backgrounds of poverty and many children who are not native English speakers as they explain the methods I discuss today.

These are on my website karenfusonmath.com under Classroom Videos and then

A. Classroom Components and Part 3 Math Talk has:

Math Talk Introduction

Grade 1 addition with regrouping invented method and

New Groups Below method

Grade 2 Subtraction with Ungrouping First

Grade 4 Expanded Notation Multiplication

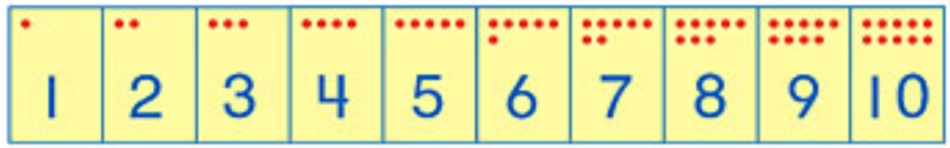
Students must learn meaningful visual representations to give meaning to mathematical words and symbols.

Here are visual supports that are powerful in helping all children understand written place value and English place value words.

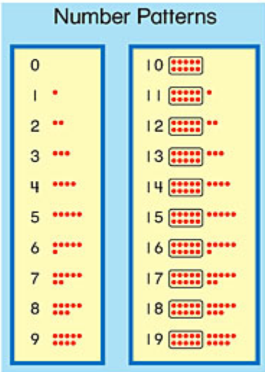
K Number Patterns in Order

Visual Supports for Patterns in Numbers and Quantities in Order

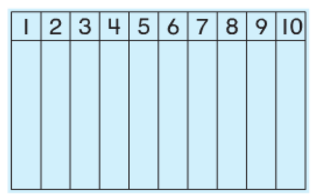
Number Parade



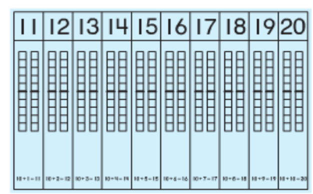
Number Patterns to 19



1-20 board



1–20 Board (front)



1–20 Board (back)

120 poster

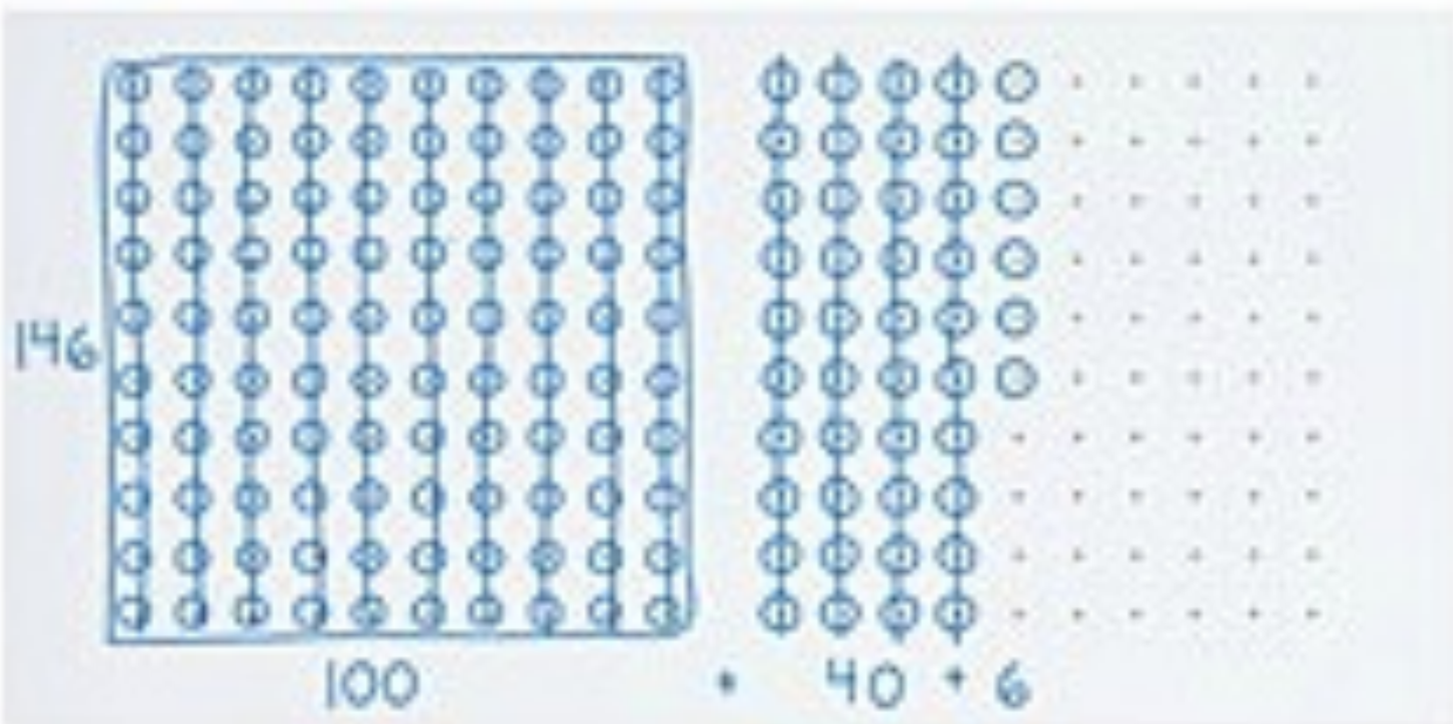
1	11	21	31	41	51	61	71	81	91	101	111
2	12	22	32	42	52	62	72	82	92	102	112
3	13	23	33	43	53	63	73	83	93	103	113
4	14	24	34	44	54	64	74	84	94	104	114
5	15	25	35	45	55	65	75	85	95	105	115
6	16	26	36	46	56	66	76	86	96	106	116
7	17	27	37	47	57	67	77	87	97	107	117
8	18	28	38	48	58	68	78	88	98	108	118
9	19	29	39	49	59	69	79	89	99	109	119
10	20	30	40	50	60	70	80	90	100	110	120

The Vertical 120 Poster

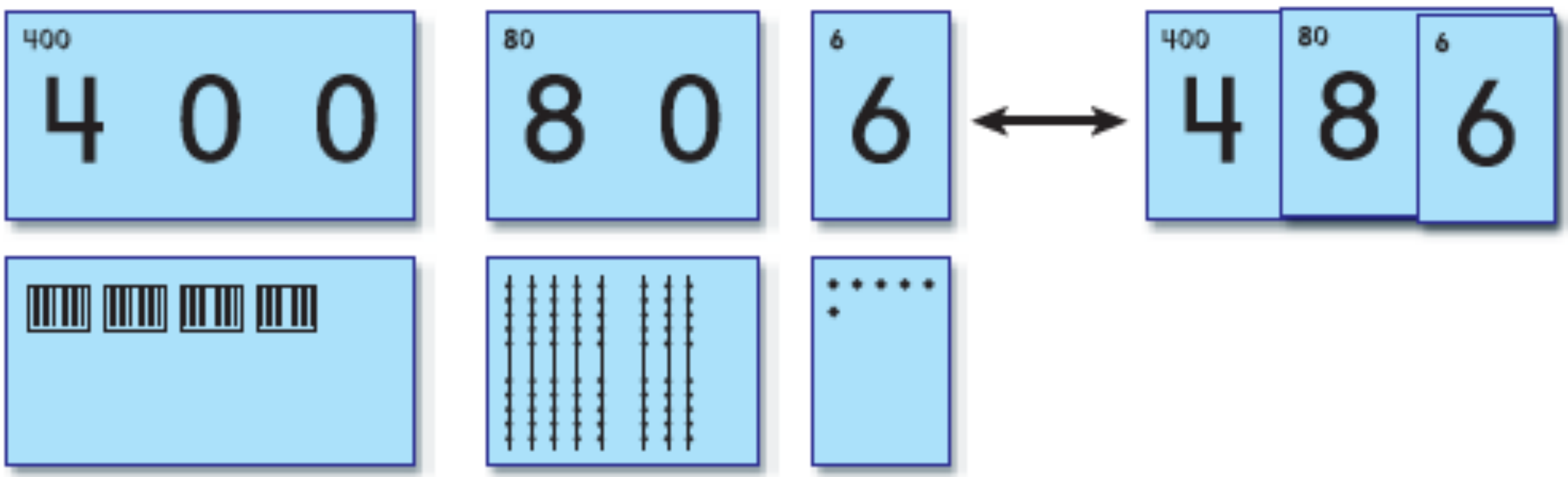
1	11	21	31	41	51	61	71	81	91	101	111
2	12	22	32	42	52	62	72	82	92	102	112
3	13	23	33	43	53	63	73	83	93	103	113
4	14	24	34	44	54	64	74	84	94	104	114
5	15	25	35	45	55	65	75	85	95	105	115
6	16	26	36	46	56	66	76	86	96	106	116
7	17	27	37	47	57	67	77	87	97	107	117
8	18	28	38	48	58	68	78	88	98	108	118
9	19	29	39	49	59	69	79	89	99	109	119
10	20	30	40	50	60	70	80	90	100	110	120

G2 Place Value Drawings 2.NBT.1 and 2.NBT.3

Hundreds **Tens** **Ones**

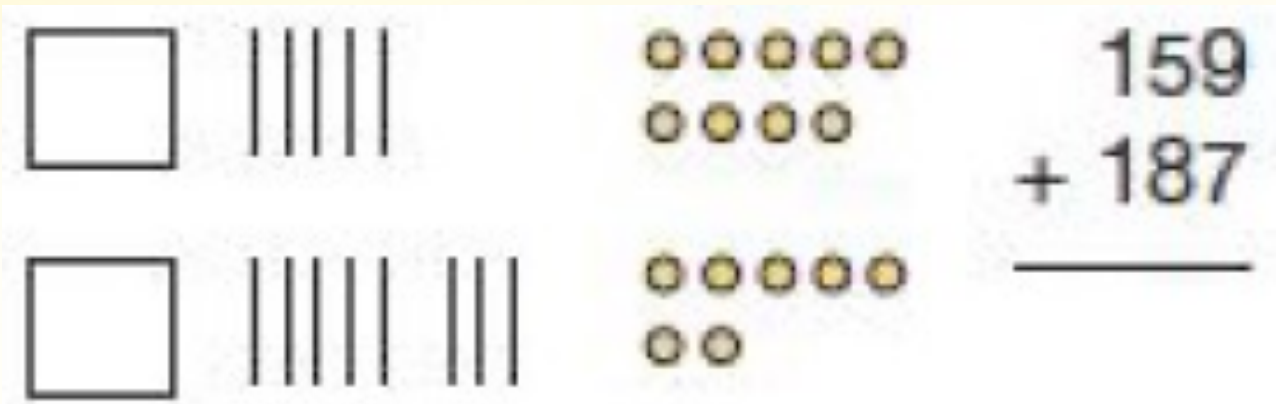


G2 Secret Code Cards for 486 2.NBT.1 and 3



New Groups Below is the best multidigit addition method.

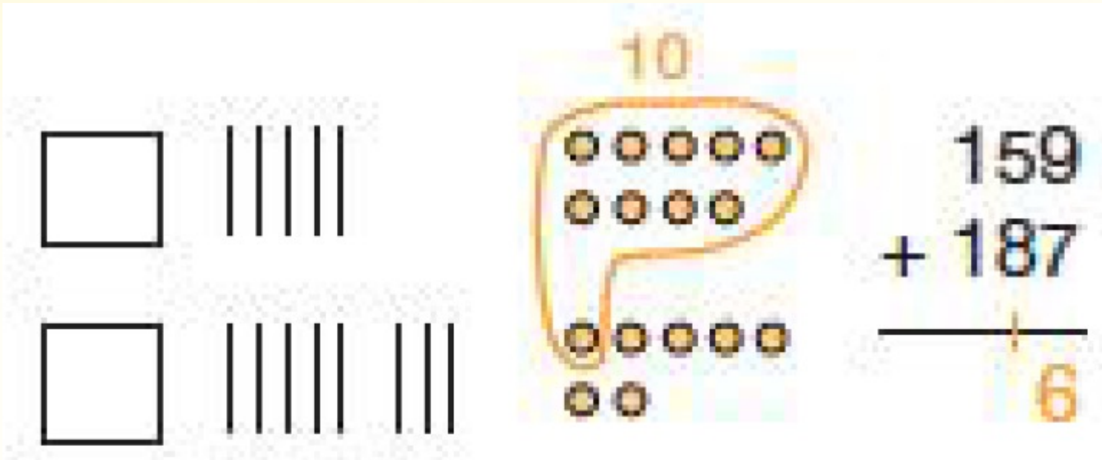
Step 1: Make and relate a place value drawing to the addition problem.



I drew one hundred, five tens, and nine ones to show one hundred fifty nine, and here below it I drew one hundred, eight tens, and seven ones for one hundred eighty seven. I put the ones below the ones, the tens below the tens, and the hundreds below the hundreds so I could add them easily.

New Groups Below is the best multidigit addition method.

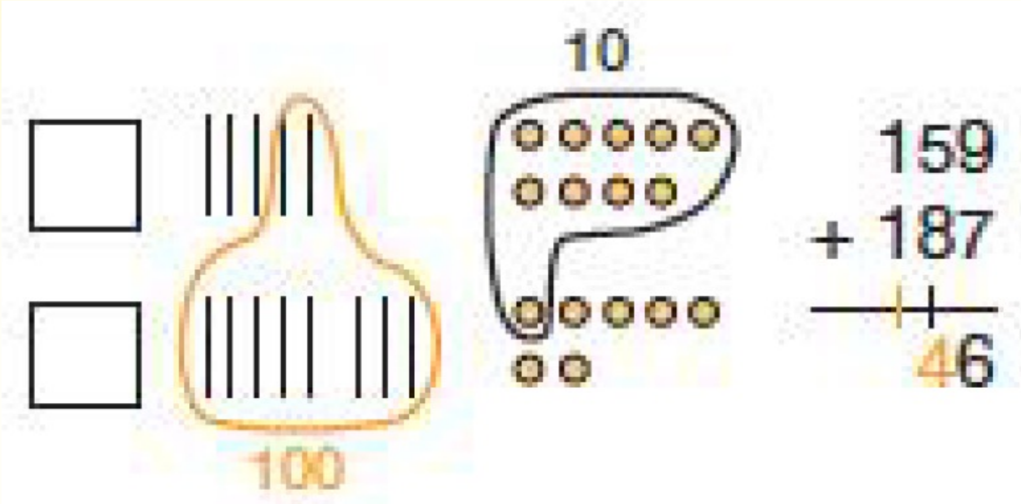
Step 2: Add the numbers in the ones place.



See here in my drawing, nine ones need one more one from the seven to make ten ones that I circled here, and I wrote 10. That leaves six ones here. With the numbers the seven gives one to the nine to make ten that I write over here in the tens column, see one ten. And I write six ones here in the ones column.

New Groups Below is the best multidigit addition method.

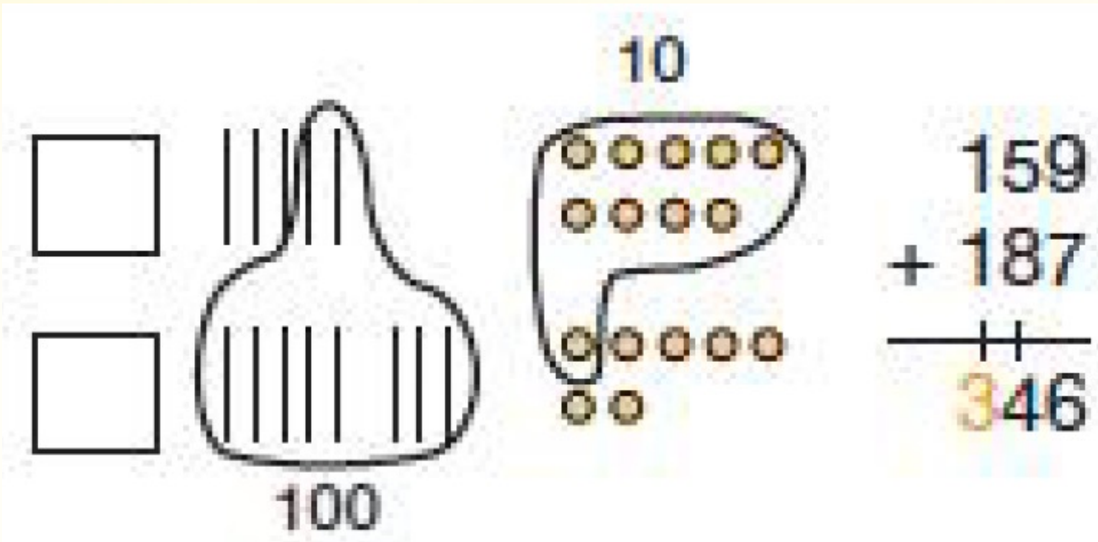
Step 3: Add the numbers in the tens place.



With the tens, I start with eight because it is more than five so it is easier. I get two tens from five tens to make ten tens, see here, and I write one hundred here to remind me that the ten tens make one hundred. There are three tens left in the five tens and I have one more ten from my ones (see here in my drawing and the one ten at the bottom of the tens column). That makes four tens and the one hundred. So in my problem I write the one hundred below in the hundreds column and the four tens in the tens column.

New Groups Below is the best multidigit addition method.

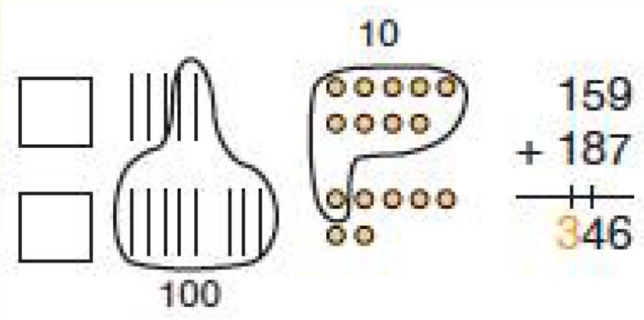
Step 4: Add the hundreds.



There are three hundreds, two in the original numbers I'm adding and one new hundred from the ten tens. I write three hundreds here in the hundreds column. Are there any questions? Yes, Stephanie.

New Groups Below is the best multidigit addition method.

Step 5: Answer questions from fellow students about the work.

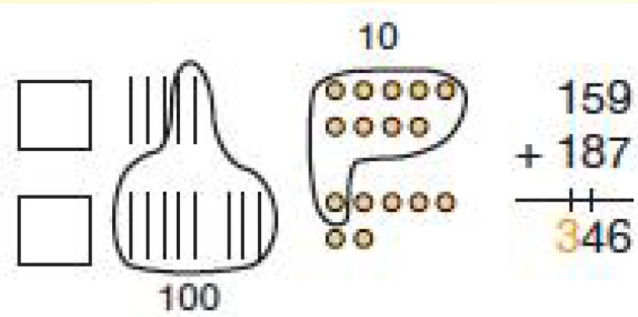


Stephanie: For the tens, you never said fourteen tens as the total of the tens. Why not?

Because when I'm making ten tens, I just can write that one hundred over here with the hundreds and just think about how many tens I need to write. But I can think eight tens and five tens is thirteen tens and one more ten is fourteen tens, so that is one hundred and four tens. You can do it either way. (Aki)

New Groups Below is the best multidigit addition method.

Step 6: Answer questions from fellow students about the work.

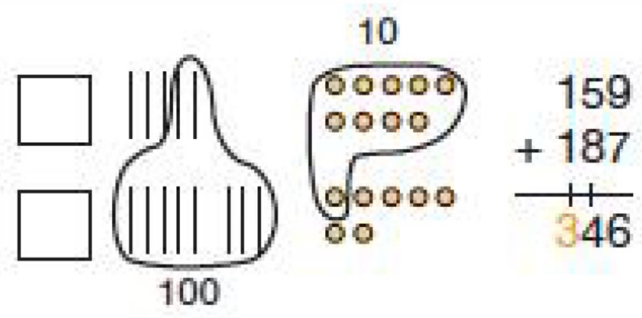


Aki: Do you still need to make the drawings or did you just make them so you could explain better?

I don't have to make the drawings, but I can explain better with a drawing because you can see the hundreds, tens, and ones so well. (Jorge)

New Groups Below is the best multidigit addition method.

Step 7: Answer questions from fellow students about the work.

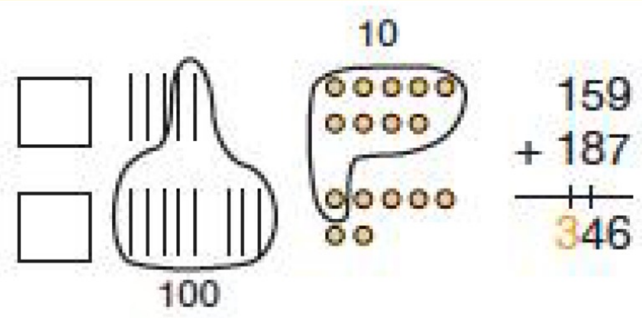


Jorge: Do you do make-a-ten in your head or just know those answers?

I just know all of the nine totals because of the pattern: the ones number in the teen number is one less that the number added to nine because it has to give one to nine to make ten. So nine plus seven is sixteen. I just know that pattern super fast. For eight plus five, I do make-a-ten fast, sort of just thinking five minus two is three, so thirteen. (Sam)

New Groups Below is the best multidigit addition method.

Step 8: Answer questions from fellow students about the work.



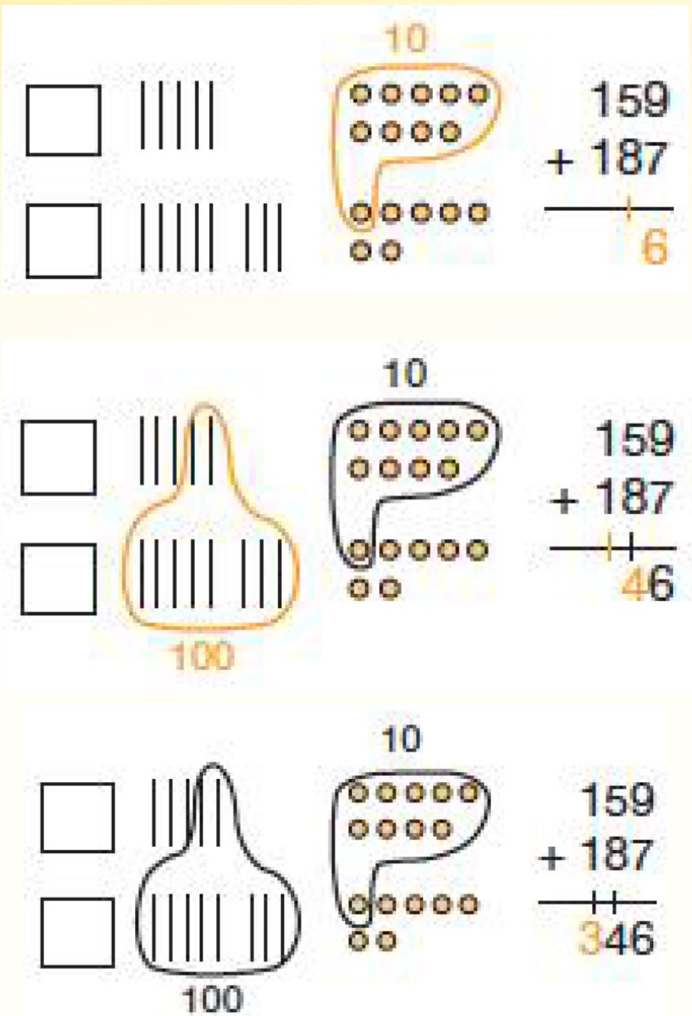
Sam: I know five and eight is thirteen, so why did you write a four in the tens column, Karen?

Because I had one more ten from the ones. See here in the drawing: nine ones and one one from the seven ones make ten ones. I wrote 10 here to remind me, and here in the problem I wrote the new one ten below where I can add it in after I find thirteen. You have to write your new one ten big enough to be sure you see it.

Sam: Oh yes, I see it now. I can see the new one ten when I write it, but I couldn't see yours.

OK, thanks. I'll write it bigger next time so everyone can see it.

New Groups Below is the best multidigit addition method.



Think about why New Groups Below is better than writing the new one ten and new one hundred above the problem. Type your answer in the chat, and I will read these to everyone.

```
  1 1
  159
+ 187
-----
  346
```

Ways in which New Groups Below is better than New Groups Above.

Variations that support and use place value correctly are crucial.

It is easy to see the teen total in New Groups Below because they are close together. For example, see the 16 ones and the 14 tens for Method E.

```
  159
+ 187
  1 1
  346
```

In the New Groups Above method these teen numbers are widely separated and difficult to see as teen numbers.

```
  1 1
  159
+ 187
  346
```

It is easy to see where to write the new unit: The 1 ten for the 16 ones is written in the column just to the left of the 6 below in the ones column and similarly for the 14 tens in the tens column.

In the New Groups Above Method, some children say that the separation of the teen numbers makes it more difficult to put the new 1 group in the next left column.

Ways in which New Groups Below is better than New Groups Above.

Variations that make single-digit computations easier.

In New Groups Below one adds the two larger numbers first: add the 5 tens and 8 tens to get 13 tens and then add the 1 new ten waiting below.

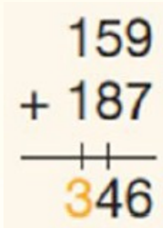
```
  159
+ 187
  11
  346
```

In New Groups Above children may forget to add the 1 new group above if they add the larger numbers first. And adding the 1 to the top number and adding that total to the second number means that the child has to add a number they do not see (6) and ignore a number they do see (5) in order to get 14 tens.

```
  11
  159
+ 187
-----
  346
```

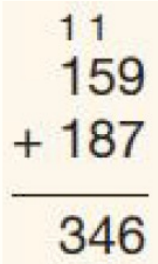
Ways in which New Groups Below is better than New Groups Above.

Variations that allow children to write teen numbers in their usual order left to right, which is the one ten and then the ones,



This is easy to do for New Groups Below (9 plus 7 is sixteen which I can write as 1 then 6).

For New Groups Above, children are often told to write the 6 and carry/regroup the 1, the opposite order to their usual way of writing numbers, which is left to right. Sometimes children have the 6 above the tens place because they wrote the 1 ten first and then the 6 ones.



Ways in which New Groups Below is better than New Groups Above.

Variations that keep the initial multidigit numbers unchanged because they are conceptually clearer:.

New Groups Below does not change the original addends 159 or 187. Each addend and the total are in their own horizontal space.

```
  159
+ 187
  11
  346
```

For New Groups Above some children object to writing 1 above the top number because they say that you are changing the problem (and you are).

```
  1 1
  159
+ 187
  346
```

This is the most typical subtraction error. Many students make this error.

$$
\begin{array}{r}
346 \\
-\,159 \\
\hline
213
\end{array}
$$

What is this error?

Why do students make it?

What can we do about it?

Here is a powerful approach to prevent this error.

$$\begin{array}{r} 346 \\ -\,159 \\ \hline 213 \end{array}$$

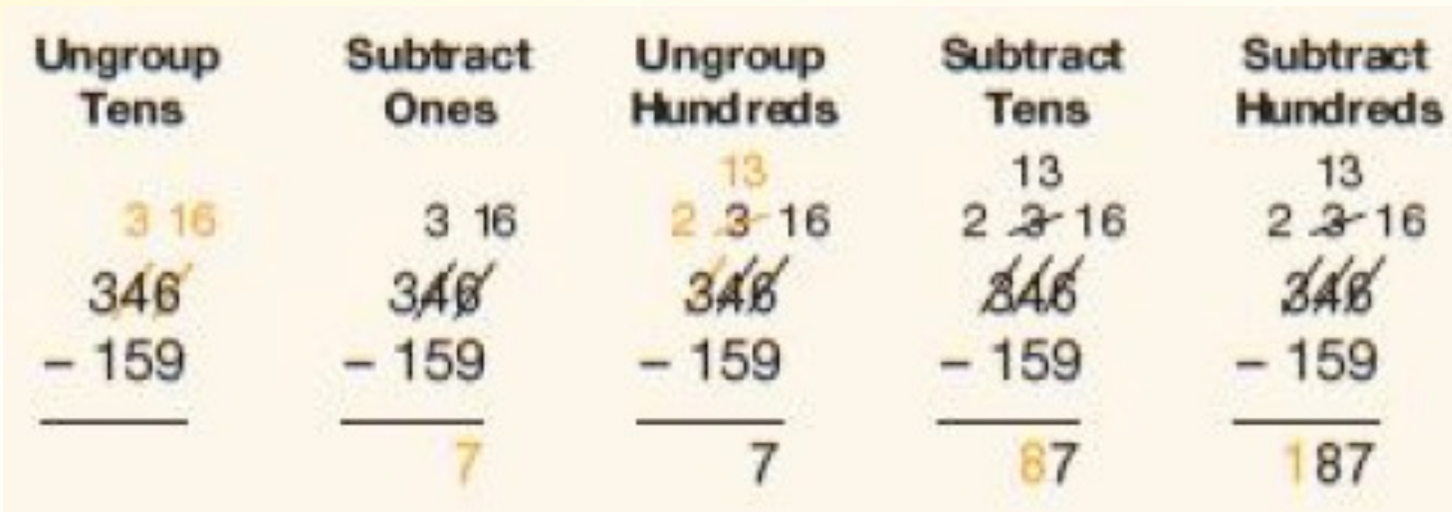
Draw attention to the total in the subtraction situation by encircling it. We call this a magnifying glass because you are looking inside the total to find its parts.

Do not draw or show the known addend 159 because it is part of 346.

The magnifying glass stops students from subtracting right away and reminds them to check each place to see if they need to get more in order to subtract in that place.

I drew three hundreds, four tens, and six ones to show three hundred forty six. I wrote one hundred,five tens, nine ones below three hundred forty six in my problem, but I did not draw it because it is already part of three hundred forty six. To subtract, we separate the total into two numbers, the number we are taking away and the number that is left. Here I drew my magnifying glass around the total to remind me to check if I need to ungroup to get more to subtract.

Another frequent error is created by the usual alternating subtraction method in which one ungoups and then subtracts.

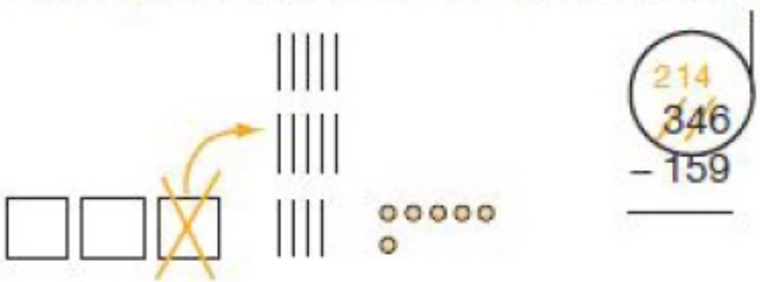


In this step students have just subtracted 9 from 16. They look left and see 3 and 5 and are in subtraction mode, so they subtract to get 2.

So the best multidigit subtraction method does not alternate ungrouping and subtracting. You do all necessary ungrouping first and then subtract in all places. Each of these processes can be done left to right or right to left.

Step 1: Check to ungroup as needed, here starting on the left.

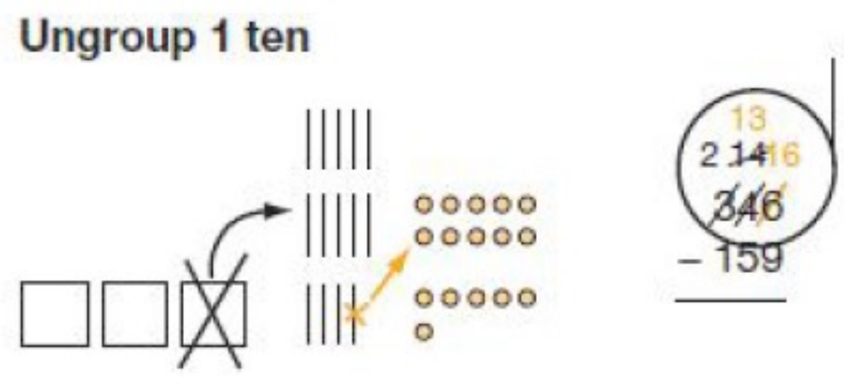
Ungroup 1 hundred (solving left to right)



I checked to see if I need to ungroup left to right. I can do it right to left too. So here in the hundreds I can take one hundred from three hundreds, so that column is OK. In the tens column, I cannot take five tens from four tens because five is more than four. So I need to get more tens to go with my four tens. I open up one hundred to make it be ten tens. Here I wrote my ten tens in two rows of five so you can see them clearly. And in my problem I showed that ungrouping by crossing out the three hundreds and writing the two hundreds I have left. And the four tens become fourteen tens here. So I'll be able to subtract the tens.

So the best multidigit subtraction method does not alternate ungrouping and subtracting. You do all necessary ungrouping first and then subtract in all places. Each of these processes can be done left to right or right to left.

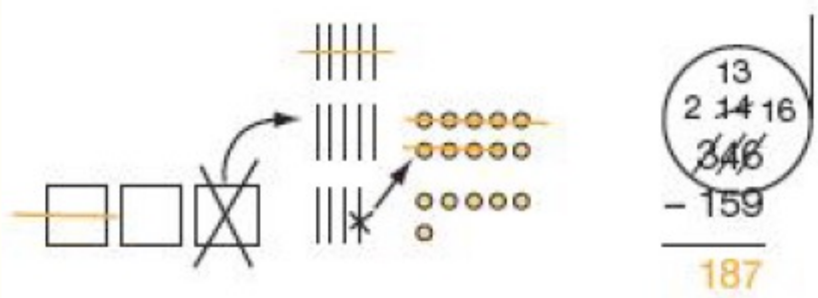
Step 2: Check to ungroup as needed, moving to the right to check the ones place.



Now I check to see if I can subtract the ones. Nope. Nine is more than six, so I need to get more ones also. I open up one ten here to show that it has ten ones hiding in it. I write them in two rows of five so I know I made exactly ten and you can see them. In my problem I ungrouped by taking one ten from the fourteen tens and writing thirteen above in the tens column. And the ten ones make sixteen ones with the six, so I write sixteen at the top of the ones column.

So the best multidigit subtraction method does not alternate ungrouping and subtracting. You do all necessary ungrouping first and then subtract in all places. Each of these processes can be done left to right or right to left.

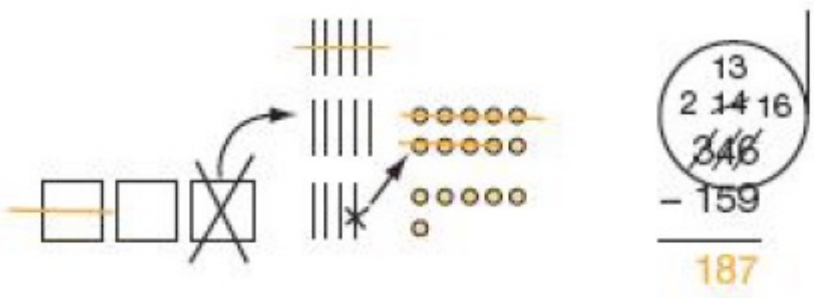
Step 3: Subtract in each place moving from left to right or from right to left.



Now I can subtract in every column. I can go in either direction. I'll go left to right again. I take away one hundred in my drawing, and one hundred is left. My problem agrees: I take away one hundred from the two hundreds and write the one hundred that is left. I'll subtract five tens from thirteen tens and get eight tens. I just know that. But here in my drawing I'll take the five tens from the ten tens, and I can do make-a-ten if I don't know thirteen minus five. See, five more left in the ten and the three in thirteen make eight. For the ones I can use Karen's pattern she just explained, that the teen total is one less than the ones added on to a nine. So sixteen minus nine is seven. See here in the drawing, you can see the one extra with the nine that gets added to the six ones to make seven ones. Are there any questions? Yes, Sybilla?

So the best multidigit subtraction method does not alternate ungrouping and subtracting. You do all necessary ungrouping first and then subtract in all places. Each of these processes can be done left to right or right to left.

Step 4: Ask and answer questions from classmates.



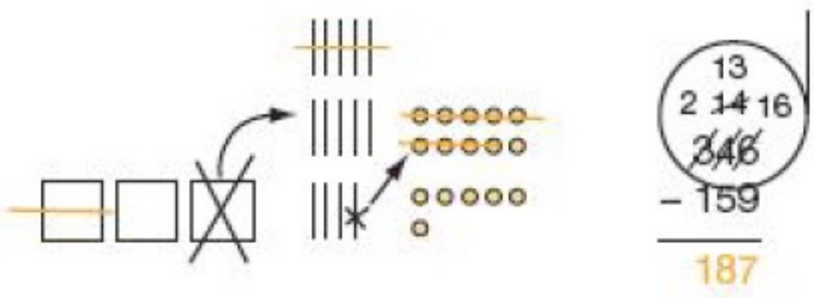
Sybilla: Doug, why didn't you subtract six ones from nine ones to get three ones in the answer?

Because we have to subtract the addend from the total. Six is part of the total, so we have to subtract from it. But we can't, so that's why I had to get more ones here. Good question, even though I know you know this. Hank?

So the best multidigit subtraction method does not alternate ungrouping and subtracting. You do all necessary ungrouping first and then subtract in all places. Each of these processes can be done left to right or right to left.

Step 5: Ask and answer questions from classmates.

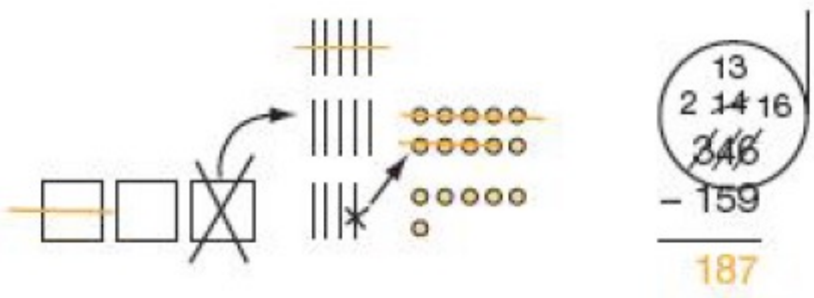
Hank: What if you checked your hundreds and the bottom number was bigger? How could you subtract?



I couldn't. The total has to be bigger than the addend I subtract because that addend is just part of the total. But sometimes I write the numbers backwards, so I check the problem again if I can't subtract the hundreds. Efrain?

So the best multidigit subtraction method does not alternate ungrouping and subtracting. You do all necessary ungrouping first and then subtract in all places. Each of these processes can be done left to right or right to left.

Step 6: Ask and answer questions from classmates.



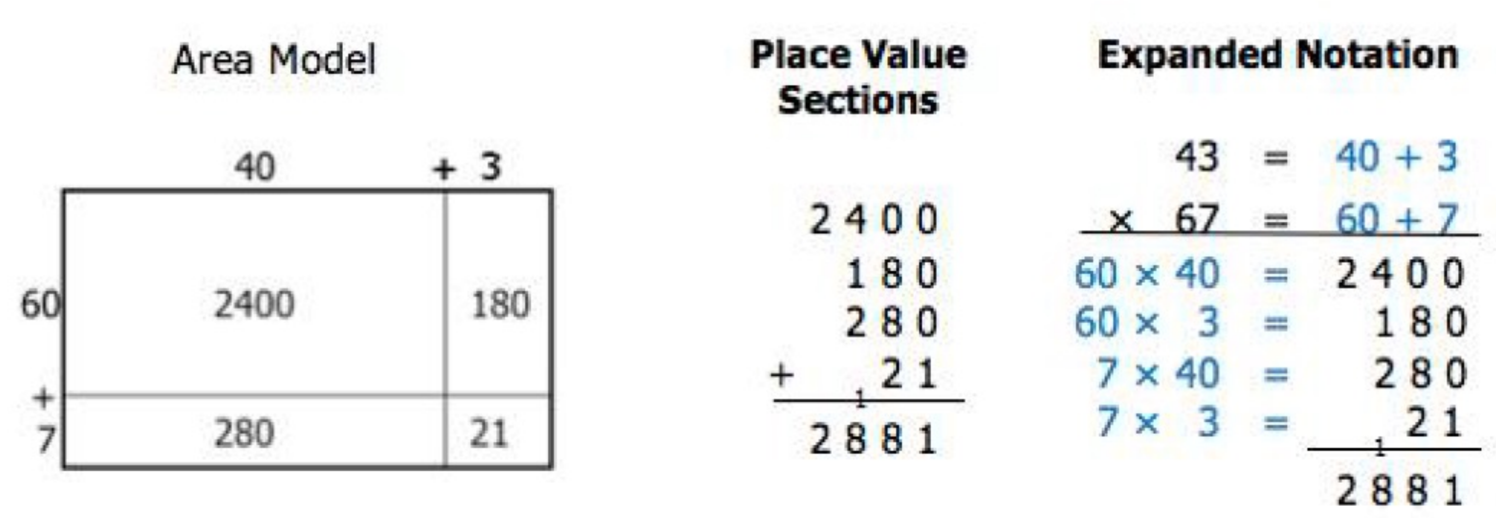
Efrain: How would your problem be different if you had ungrouped right to left?

Only the tens place would look different. Remember how we did it both ways and talked about this yesterday? And look at Yeping's problem. He ungrouped right to left. The tens place looks different because you ungroup one ten to make ten ones before you get ten tens. So you write three and then thirteen. But I end up with thirteen, so the ungrouping gets the same number in each column ready to subtract.

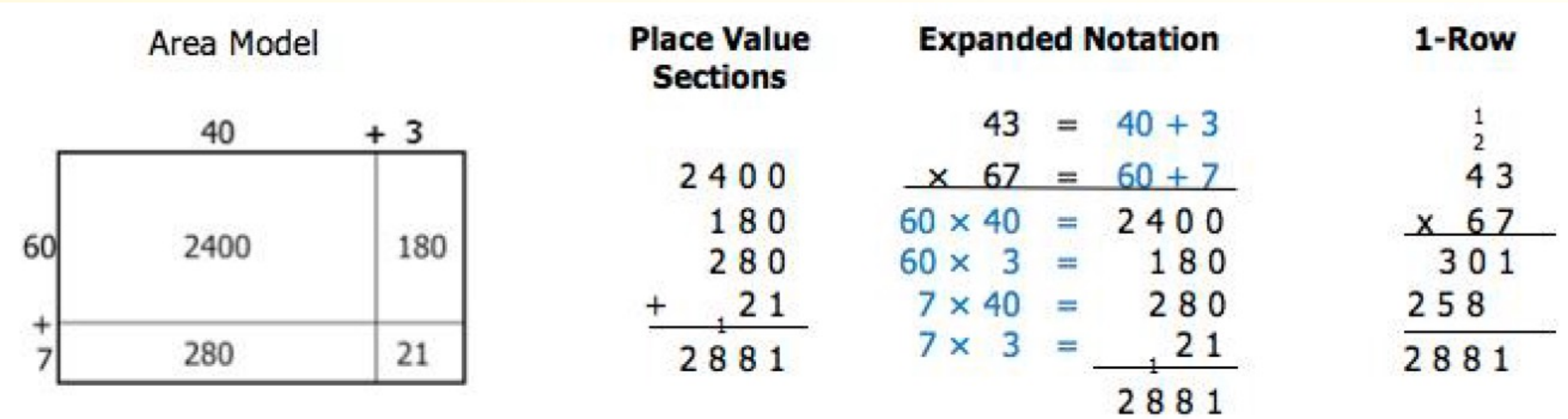
Expanded Notation is the best multidigit multiplication method.

The steps in blue can be dropped whenever students are ready. Then the partial products can be written under the factors.

Some students cannot handle the complex lay-out of expanded notation, but they can make and understand an area model and add all of the partial products as shown in the Place Value Sections method.



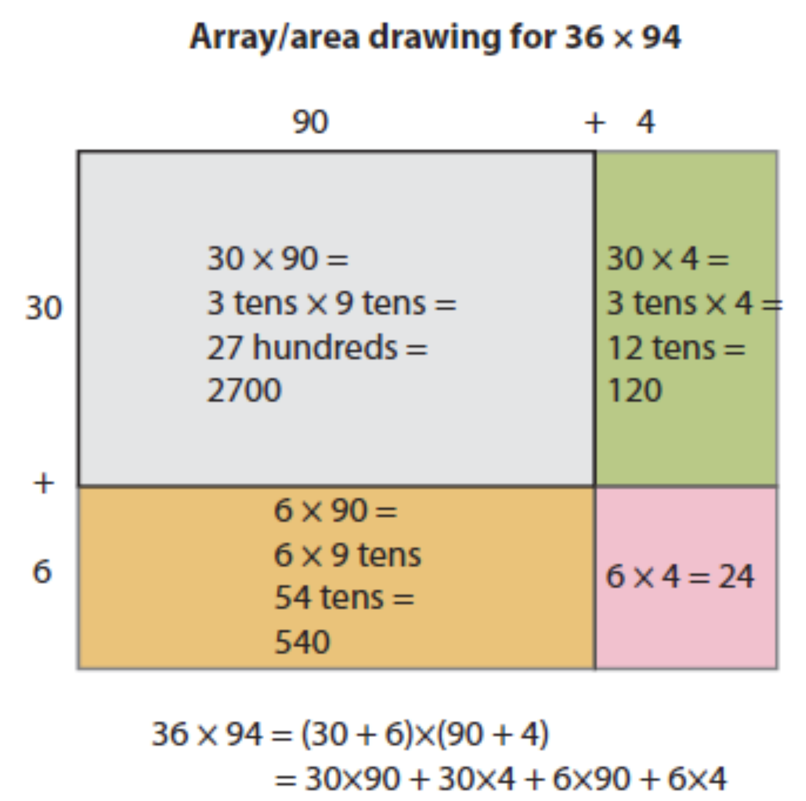
The 1-row method on the right is taken to be "the standard algorithm". But see how it uses wrong place-values for the step 60 x 3 = 180 but the 1 hundred is written above the tens place.

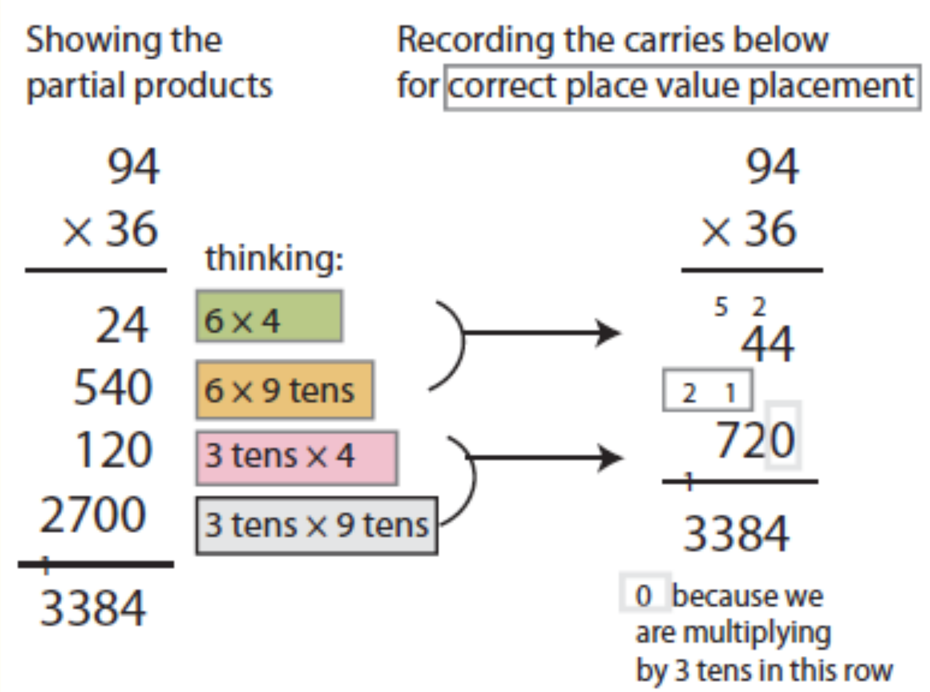


Notice how the area model is helpful for all methods to see what place in one factor gets multiplied by what place in the other factor.

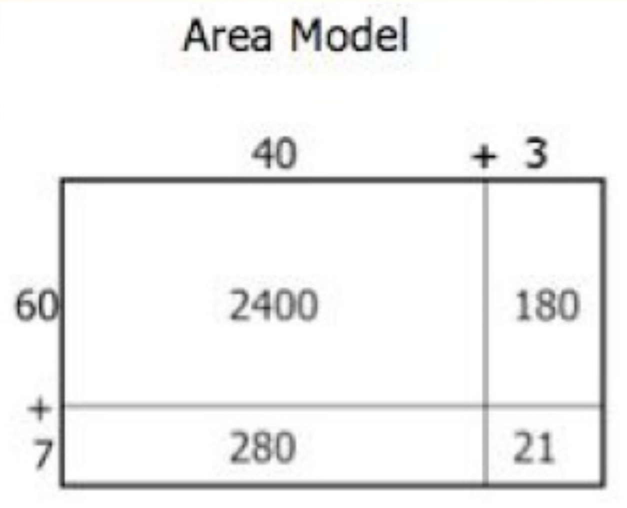
Fuson and Beckmann show a way to write the 1-Row method using correct place values by writing the products below the factors.

Fuson and Beckmann show a way to write the 1-Row Method using correct place values by writing the products below the factors. This example also shows that multiplying can begin with multiplying by the ones first although if you begin from the left as in the Expanded Notation method, other products can be aligned under the first largest product and some student find this easiest.

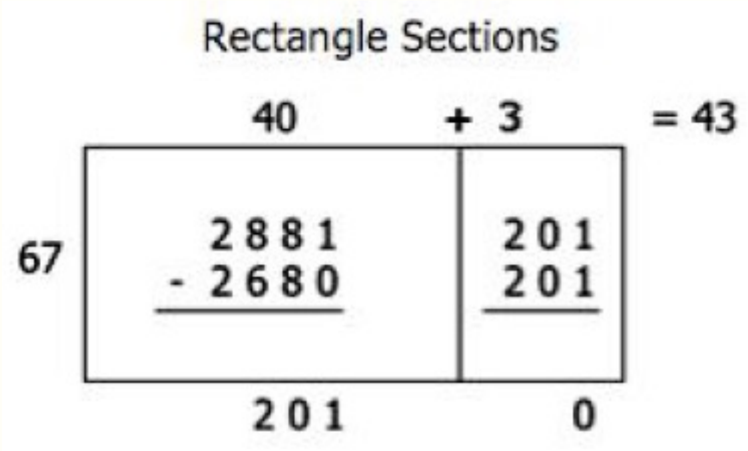




Relationships between multidigit multiplication and multidigit division are important. The area model can be used for both operations with division seen as finding the place values 40 and 3 in the unknown factor along the top of the rectangle.

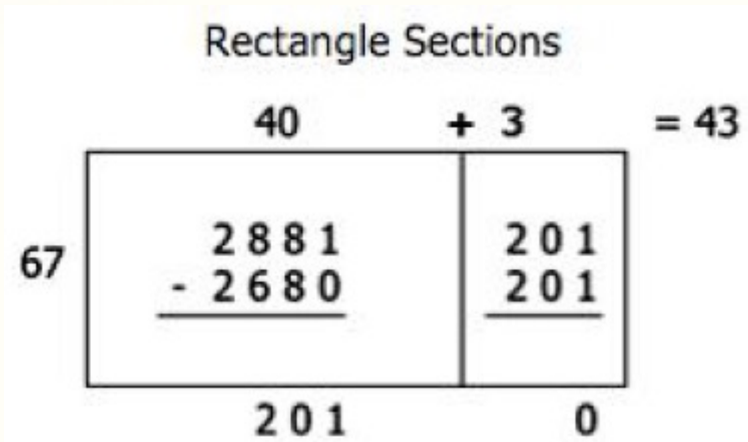


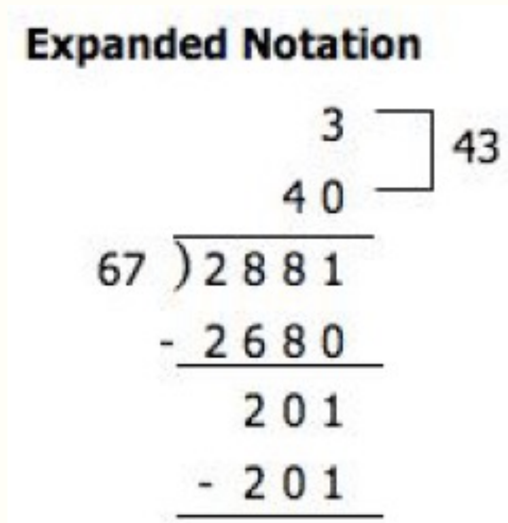
67 x 43 = 2881

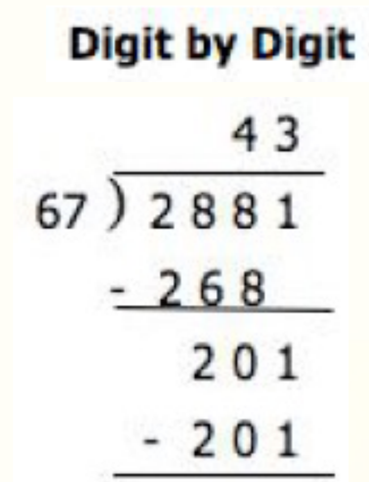


67)2881 = 40 + 3 = 43

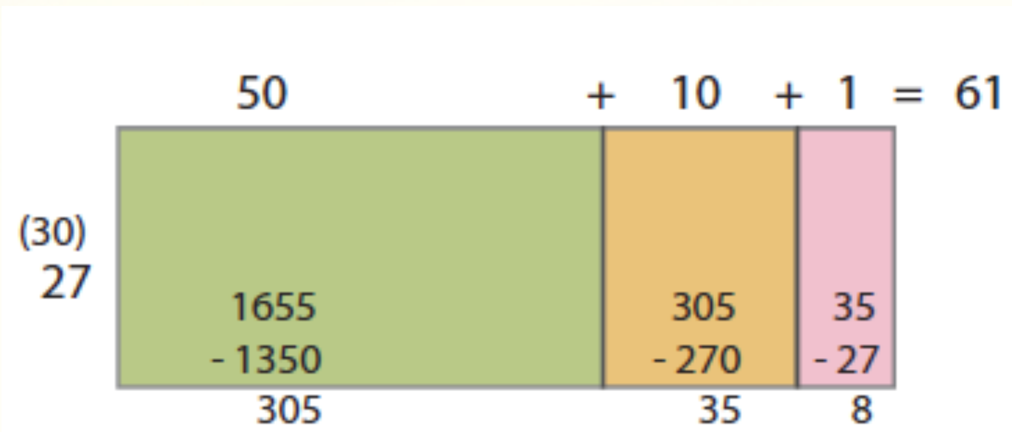
The Rectangle Sections drawn model can be used as a written method or it can be related to the Expanded Notation or the Digit by Digit method. The Expanded Method shows place values for each step, so many students can understand it better.

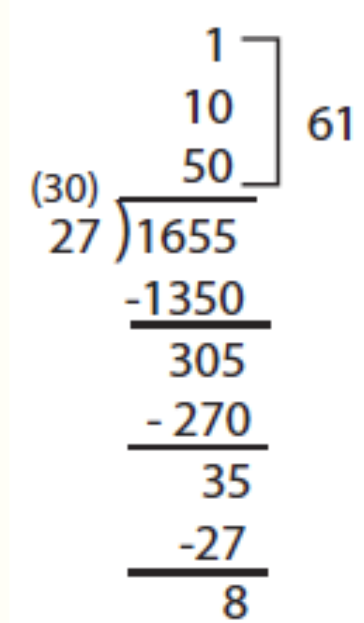






Multidigit division can be difficult if students feel they have to write the exact multiplier at each place value. Fuson and Beckmann show for the Rectangle Sections and Expanded Notation methods how students can underestimate a multiplier and keep adding on a partial product without erasing work already completed.





Rounding 27 to 30 produces the underestimate 50 at the first step, but this method allows the division process to be continued.

Summary of multidigit methods:

New Groups Below and Ungroup As Needed First are the best multidigit addition and subtraction methods. They involve minor but crucial changes in the more difficult misleading forms of the standard algorithm.

Expanded Notation are the best multiplication and division methods.

These methods are not new. They are supported by major professional organizations and are in publications for teachers by NCTM and NCSM.

It is time for you to make the lives of your students better!!! These methods can be developed by students when they first engage with a multidigit operation if they are using drawings.

Drawings are crucial!!! Students need to make them and relate them to their thinking and explanations. A nurturing Math Talk Community is important for the teacher and for students.

On my website karenfusonmath.com are publications describing these methods, related methods, and the research.

The Best Multidigit Computation Methods: A Cross-cultural Cognitive, Empirical, and Mathematical Analysis, Karen C. Fuson. Universal Journal of Educational Research 8(4): 1299-1314, 2020 DOI: 10.13189/ujer.2020.080421

Fuson, K. C. & Beckmann, S. (Fall/Winter, 2012-2013). Standard algorithms in the Common Core State Standards. *National Council of Supervisors of Mathematics Journal of Mathematics Education Leadership, 14 (2)*, 14-30. Also at http://www.mathedleadership.org/docs/resources/journals/NCSMJournal_ST_Algorithms_Fuson_Beckmann.pdf

Mathematical Practices

Math Sense-Making	Math Structure	Math Drawings	Math Explaining
Make sense and use of appropriate precision.	**See structure and generalize.**	**Model and use tools.**	**Reason, explain, and question.**
MP1 Make sense of problems and persevere in solving them. **MP6** Attend to precision.	**MP7** Look for and make use of structure. **MP8** Look for and express regularity in repeated reasoning.	**MP4** Model with mathematics. **MP5** Use appropriate tools strategically.	**MP2** Reason abstractly and quantitatively. **MP3** Construct viable arguments and critique the reasoning of others.
Teachers continually assist students to do math sense-making about math structure using math drawings to support math explaining.			

Teachers continually assist students to do **math sense-making** about **math structure** using **math drawings** to support **math explaining**.

Number Talks are not a Math Talk Community

Number Talks can be a helpful introduction to children describing their thinking. But they are best used briefly and then followed by building a Math Talk Community in the regular math classroom.

It is not necessary to use Number Talks before building a Math Talk Community. It is necessary to help children have some way of representing the math topic to be discussed. Math drawings are windows into minds, and they help everyone understand the Math Talk.

Number Talks have these equity issues:

Only mental methods can be used so problems are often very easy.

Mental methods bias students toward counting on methods that do not generalize easily to larger numbers.

Students do not make drawings, so it is difficult for other students to see their thinking.

Students have to describe their methods in words. This is difficult for some.

Students do not write their own methods. The teacher writes their methods, implying that only the teacher can write or explain a method.

The talks are done in a separate part of the classroom away from regular math class. So what is happening in math class? Are students understanding, drawing, and explaining there? If so, why are Number Talks needed?

Some teachers who do Number Talks think that they have done Math Talk and teach the regular math class as a teacher demonstration.

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