math expressions

A Nurturing Math Talk Community in Math Expressions

BUILDING A NEW STANDARD OF SUCCESS

Heinemann

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WHAT IS A NURTURING MATH TALK COMMUNITY?

High-level state standards expect students to reason about mathematical ideas and to discuss and explain that reasoning. Visual representations (e.g., diagrams) are used to support understanding. Steps in written methods can be supported by steps in visual representations that students draw. The eight mathematical practices also involve reasoning and explaining and using drawings.

Math Expressions™ supported these emphases even before the Common Core State Standards (CCSS) existed because these emphases came from research used in designing *Math Expressions* and the CCSS. For example, the National Council for Teachers of Mathematics (NCTM®) Standards and two National Research Council reports (*Adding It Up: Helping Children Learn Mathematics and How Students Learn: Mathematics in the Classroom*) emphasize the need for students to discuss their mathematical thinking as a way to increase understanding. The CMW Research Project that developed the *Math Expressions* program carried out research to identify crucial aspects of such discussions and to identify levels through which teachers could transition from traditional teacher-focused instruction to productive student-to-student discussion, monitored and supported by the teacher, as recommended by the National Research Council reports. We also sought in this project to identify other features that could support effective discussions.

We found that the term Math Talk was effective in communicating with teachers and students to achieve the focus on discussion we desired. Our vision was for Math Talk to be an instructional conversation directed by the teacher but with as much student-to-student talk as possible. Math Talk is focused on developing the understanding of all students in the class and on moving everyone along the learning path for the topic.

Math Talk needs to occur in a safe and supportive classroom environment if learning is to occur. We describe such an environment as a Nurturing Meaning-Making Math Talk Community focused on teaching/learning by the students and the teacher. The rest of this paper describes this community in more detail.

THE SOLVE AND DISCUSS STRUCTURE

We found that two kinds of Solve, Explain, Question, and Justify (often shortened in use to Solve and Discuss) classroom activity structures were very effective in engaging all students in Math Talk (see Table 1). In both structures, all students solve problems simultaneously. In the first structure, as many students as possible go to the board to solve a problem while the rest of the students work at their seats. Then the teacher selects two or three students from the board who have interesting solutions, or need the chance to explain their work, to talk about their solution. Only two or three students need to explain their work because students usually

Table 1

Solve and Discuss	
1	**Solve:** All students solve.
2	**Explain:** One student explains and then asks, "Are there any questions?"
3	**Question:** Other students ask questions to clarify or extend.
4	**Justify:** The original explainer responds to the questions by explaining more (justifying the original explanation).

Any student at any time can ask for help from anyone. Typically another student explains, so the class loops through 2, 3, and 4 again. The discussion can now also contrast and compare the first and second solutions as well as others in the past.

cannot maintain concentration for more than two or three discussions of the same problem. Next, a different group of students goes to the board to solve the next problem. This process is very motivating. Most students enjoy solving problems at the board even if they do not get the chance to explain their work. While the students are working at the board, the teacher has a chance to see how solutions evolve. The teacher also gets a good sense for how individual students are doing. In one class period, many or even all of the students can get a turn at the board.

The second effective classroom structure allows every student in the classroom to explain his or her solution. Every student solves a problem at his or her seat. Then two or three students are selected by the teacher to go to the board to draw their solutions. The students left at their desks then pair up and explain their solutions to each other. Then the class discusses the solutions of those students at the board. Students at their seats can write their solutions on paper, which can be picked up and skimmed later by the teacher to see how students are progressing. Another option is to have students at their desks solve problems on the large (12" by 19") individual *Math Expressions* dry-erase boards called MathBoards. These MathBoards permit the teacher to send any additional student to the board for another explanation of the discussed problem because the drawings on the MathBoard are large enough to be seen by classmates. An important feature of both of these classroom structures is that no class learning time is lost. In other approaches, when students are sent to the board to draw their work, the rest of the class remains at their seats doing nothing. In the case of these *Math Expressions* structures, the students at their desks are just as involved in the problem solving as those at the board. Sometimes a step-by-step variation of these activity structures is helpful. Teachers can have each student explain one step of a solution at a time until a final solution is reached, clarifying as needed.

Another method is to put students in pairs. The pairs can solve together and explain their work, with the less advanced student explaining first and the more advanced student expanding.

EVERYONE ASSISTS LEARNING

Table 2 (below) shows ways in which everyone in the class can assist the learning of others. Engaging and involving includes inviting all students to share ideas and questions, promoting analysis and discussion, and expecting that all students participate in developing understanding together in the community. Managing includes helping students monitor, be responsible for, and take ownership of their own learning. Initially the teacher engages and involves and also manages, but with encouragement and support students can also assist the Nurturing Math Talk Community in these ways. Coaching involves two major categories. In show/explain classroom members may model as a student or instruct/explain as a teacher. In focus and extend, the teacher or student coach may cognitively structure and clarify, question, or give feedback. These specific means of assisting learning occur as students engage in the CCSS mathematical practices shown in Table 3. They lead attention that enables everyone to connect visual, verbal, and gestural aspects of teaching/learning.

Table 2

Responsive Means of Assisting Learning by Teacher and Students		
Engage and Involve **Manage** **Coach** →	Show/explain • Model (as a student) • Instruct/explain (as a teacher)	Focus and extend • Cognitive structure and question • Question • Give feedback

STRUCTURING MATHEMATICAL PRACTICE STANDARDS

The mathematical practice standards can be collapsed in pairs and put into a meaningful, memorable sentence that reminds teachers of their teaching task. The pairs are shown in Table 3 with labels above each pair. The summarizing sentence that uses all of these pairs is at the top of the table. Teachers have said that they find this sentence very useful in teaching the practice standards.

LEVELS AND COMPONENTS OF MATH TALK

Our research identified five components and four levels in the development of the Math Talk Learning Community. These are shown in Table 4. The five components of Math Talk are teacher role, questioning, explaining math thinking, mathematical representations, and student responsibility within the community. Considering these crucial components is helpful to teachers as they think about building their Nurturing Math Talk Community. It takes time for teachers and students to learn how to function well within a Nurturing Math Talk Community. We describe in Table 4 four levels that describe the functioning of such communities. These have been helpful to teachers wanting to move to higher levels of functioning. There are many supports within the *Math Expressions* Teacher Editions, such as Math Talk boxes and teacher questions, that provide specific help with Math Talk. Each unit begins with a mini-lesson about the Nurturing Math Talk Community that helps teachers and students work on improving their interactions and mathematical discussions.

LEARNING MATH TALK

Effective Math Talk cannot be implemented in a classroom overnight. A teacher must work his or her students up to Level 3 Math Talk over time. It often takes two or three months to build a classroom up to Level 3 if students are not familiar with Math Talk from the start. Initially, the teacher and more advanced students will do a lot of modeling of solving and explaining for other students. In the beginning, teachers also concentrate on building listening skills by asking students to repeat a problem, question, or explanation in their own words. Teachers also elicit questions from students. Questions may apply to any topic (e.g., How does your method relate to the method that Sam just explained? or Why did you use this method?) or may apply specifically to a given math topic. In the latter case, the teacher needs to model some of these questions to use new math vocabulary, though often more advanced students can also think of such questions. Students need to learn to stand beside their drawings and numerical work and point to parts of it with a pointer as they explain. Students

Table 3

Common Core Mathematical Practices Used in a Nurturing Math Talk Community

Teachers continually assist students to do **math sense-making** about **math structure** using **math drawings** to support **math explaining**.

Math Sense-Making

1 Make sense of problems and persevere in solving them.

6 Attend to precision.

Math Drawings

4 Model with mathematics.

5 Use appropriate tools strategically.

Math Structure

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

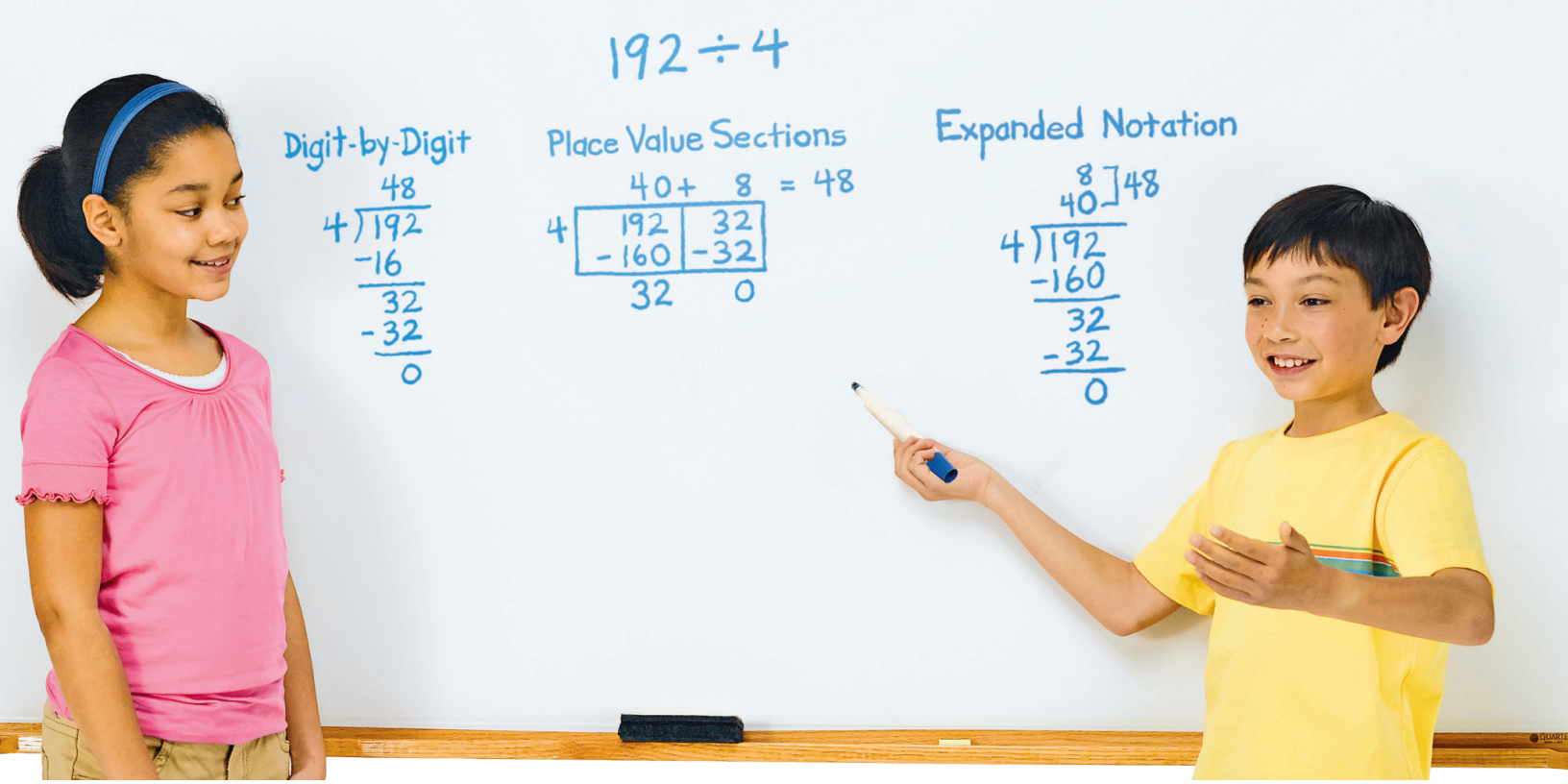
Math Explaining

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

Table 4

Components and Levels in the Math Talk Community					
	Teacher Role	**Questioning**	**Explaining Mathematical Thinking**	**Mathematical Representations**	**Student Responsibility Within the Community**
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher's questions focus on correctness. Students provide short answer-focused responses. Teacher may tell answers.	Representations are missing or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
Level 1	Teacher encourages sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher's questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to depict their mathematical thinking.	Students feel their ideas are accepted by the classroom community. They begin to listen to each other supportively and to restate in their own words what another student said.
Level 2	Teacher facilitates conversation between students and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so others are able to follow their mathematical thinking.	Students believe they are math learners and that their ideas and the ideas of classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answer with little prompting from the teacher.	Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.	Students believe they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same.



often initially explain only one part of their thinking or explain it incompletely. Using questions can help expand a student's explanation. The teacher or another student may also expand or clarify the student's explanation through questions while always checking with the original explainer to be sure that the added information is what the student intended to convey. In addition to helping students learn Math Talk methods, teachers often need to adjust to the Math Talk structures themselves. They must learn to wait patiently and use a "bite your tongue" strategy to allow student talk to emerge. They must also physically move to the side or back of the room to facilitate student-to-student talk so that the explainer looks at classmates and not at the teacher. From the side or back of the room, a small gesture can be used to remind the explainer to look at classmates rather than at the teacher. Teachers can provide community assistance by asking explainers if they need help, but they also need to allow wait time before doing so. Shy students initially may need the presence of a friend at the board with them, even if the friend does not help with the explaining. As teachers provide the space and support for students' voices to emerge, they often report being amazed by the mathematical thinking their students are able to express.

Talking must be safe for teachers and students. Initially the teacher needs to emphasize that Math Talk is not a test. It is helping everyone learn more by sharing your thinking. It is vital that the teacher emphasize: No making fun of anyone ever. The teacher does not have to know the answer to every question immediately. If no one including the teacher can answer a question or be clear about something, model extended problem solving by saying: "This is tricky. I need to think more about that. Let's all think about it and talk about it tomorrow." Talking to other teachers is a good way to get help for a tricky issue.

A NURTURING MEANING-MAKING MATH TALK COMMUNITY

The Math Talk environment is a helping community where inquiry and meaning-making are nurtured, and everyone is a teacher and a learner. This creates a secure base for learning and for Math Talk. It enhances everyone's mathematical understanding, competence, and confidence. Teachers build the Helping Community daily by helping students learn how to support each other at the board, in pairs or groups, or when working individually. *Math Expressions* was developed through ten years of

intensive classroom research in many classrooms with students from many different cultural and linguistic backgrounds. The Nurturing Meaning-Making Math Talk Helping Community enables students from all backgrounds to bring their family and cultural experiences into the classroom and be validated, affirmed, and understood.

The Math Talk approach used by *Math Expressions* supports deeper understanding and more complex language learning than other reform approaches, because the *Math Expressions* program provides research-based learning paths that move all students forward. Students develop the prerequisite understandings so that they can invent interesting methods. Research-based, accessible strategies are taught so that everyone has an effective method. These connect to common methods so that all methods can be discussed and related. Many students learn, connect, and explain multiple methods. Math drawings enable students to solve and explain more effectively and enable listeners to understand and question more effectively. This process of discussing the whole developmental range of solution methods permits differentiated instruction to occur in whole-class activities, but also enables students to move forward to a mathematically desirable and effective method. *Math Expressions* truly supports teachers as they develop a Nurturing Meaning-Making Math Talk Inquiry Classroom.

The opportunity for all students to explain their math thinking over time is especially valuable for students learning English, as well as for native speakers advancing their verbal communication skills. Ultimately, developing understanding and verbal communication will aid all students in their future education and careers. In addition to verbal communication, the use of math drawings is central to Math Talk. Math drawings can show the quantities in a computation and relate them to a written numerical method, or can show the situation in a word problem. The math drawings help everyone understand the student's math thinking. The special learning supports on the MathBoards enable students to learn to make meaningful drawings rapidly and then use the open space to create their own math drawings.

QUESTIONS

Questions from the teacher and from students play a crucial role in the Nurturing Math Talk Community. These questions can be genuine questions or they can be "teacher-y" questions asked to extend the learning of others. Many students love to develop and ask such questions. Questions can be general or can apply to specific math concepts. Some questions are shown in Table 5. The teacher initially may model many of these questions, but with support and encouragement many students can ask such questions independently. This increases the depth of functioning of the Math Talk Community. Posters of questions can be made and hung on the wall. A blank poster put up for each new unit can encourage students to think of and add questions that focus specifically on that unit.

Table 5

Building a Nurturing Math Talk Community

As students actively question, listen, and express ideas, they increase their mathematical knowledge and take on more responsibility for learning. Use the following types of questions as you build a Nurturing Math Talk Community in your classroom. Also use the Solve, Explain, Question, and Justify classroom structure frequently.

Elicit student thinking

- So, what is this problem about?
- Tell us what you see.
- Tell us your thinking.

Support student thinking

- Tell us more about _____.
- What were you thinking when you decided to _____?
- Use wait time: Take your time. We'll wait.

Support students to relate math drawings to steps in written methods

- Show us that written step in your drawing.
- How did your drawing tell you what to write in your written method?
- Do you have a question about any steps in the method _____ just did?
- Can you explain the _____ step in your own words?

Extend student thinking

- Restate: So you're saying that _____?
- Is that true for all cases?
- How is your way of solving like _____'s way?
- How is your way of solving different from _____'s way?

Increase participation of other students in the conversation

- Prompt students for further participation: Would someone like to add on?
- Ask students to restate someone else's reasoning: Can you repeat what _____ just said in your own words?
- Do you agree or disagree, and why?
- Does anyone think of this problem in a different way?
- Does anyone have a different answer? Will you explain your solution to us?
- Can you help your classmate explain more clearly?

CLOSING COMMENTS

Having students make math drawings to support and show their thinking has been emphasized throughout. These drawings also support student explaining and help classmates and the teacher understand the explainer's thinking. Making math drawings was emphasized in a shortened and updated adaptation of the Hufferd-Ackles, Fuson, and Sherin (2004) article by having a column "mathematical representations" in the updated table (Hufferd-Ackles, Fuson, & Sherin, 2015). Students follow a progression from learning to create math drawings that depict their math thinking to labeling their math drawings so others are able to follow their math thinking to suggesting edits in others' math drawings. Table 4 is this more recent version.

The term Nurturing Math Talk Learning Community in both papers was used to emphasize that each member of the learning community, including the teacher, learned mathematics when Math Talk was nurtured and developed. This community may be a bit different each year because each group of students may develop a math conversation culture specific to that group. Particular questions will become part of a given classroom culture and can be asked across topics. Teachers can support students by making their own question table as they get started and find questions that work well with their classroom community. Then they can post a student version in the classroom as a reference.

You can watch a webcast about developing a Math Talk Community and watch such classrooms in action at these web locations:

Teaching Progressions Watch Math Talk Community (Part 1):
karenfusonmath.com and karenfusonmath.net

Teaching Progressions Watch Math Talk Community (Part 2):

karenfusonmath.com and karenfusonmath.net

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Most papers with Fuson as an author can be downloaded from Publications at **karenfusonmath.com** or **karenfusonmath.net**.

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