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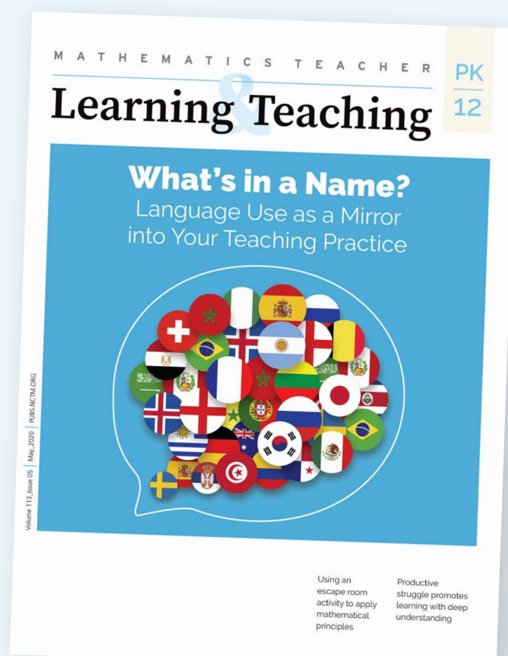

NCTM
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

BUILDING EQUITABLE Math Talk CLASSROOMS

The power of Number Talks and extensions that can build to an equitable Math Talk Classroom

Karen C. Fuson and Steve Leinwand

"**Done right, there is** something so engaging, so mathematically rich and yes, even so magical, about a number talk. Pose a straightforward exercise or simple word problem, give students time to solve it mentally, and then open the floodgates of thinking and reasoning with little more than such questions as 'How did you get your answer?' 'Who did it differently?' and 'How are our approaches similar and different?'" (Leinwand foreword to Parrish & Dominick, 2016).

We begin with our enthusiastic support for Number Talks *done right.* Many teachers around the country are doing Number Talks in their classrooms. This powerful and accessible technique can open teachers' eyes to how their students talk about their own thinking and can help students see themselves as mathematical thinkers and describers of their own thinking. Beyond student explanations and discussions of alternatives, teachers use annotations of student thinking to visually capture different ways that students arrive at both correct and incorrect solutions, opening the door to powerful discussions of conceptions and misconceptions (Humphreys & Parker, 2015; Parrish, 2010; Parrish & Dominick, 2016; Parker & Humphreys, 2018).

Imagine a third-grade class engaged in a Number Talk about finding the difference of 41 and 28. Sitting on the rug around the teacher and a flip chart, and using only mental mathematics for 30 to 45 s of individual think time, nearly all the students show engagement on their faces. In response to "Who can share with us their thinking and the solution?" it is not uncommon to hear such explanations (carefully annotated on the flip chart by the teacher) as the following:

- "I changed the 41 to 4 tens and 1 one, but because there were 8 ones in 28, I changed 4 tens and 1 one to 3 tens and 11 ones. Taking away 2 tens and 8 ones left me with 1 ten and 3 ones or 13."
- "I just counted on from 28 to 38, which is 10 more, and then from 38 to 41, which is three more, to get a difference of 13."
- "I counted on too but did it a different way. Twenty-eight and two more is 30, and then 10 more is 40, and one more means I added 2 + 10 + 1 to get 13."
- "I didn't like either 41 or 28, so I added two to both numbers to get 43 and 30 and knew that they were 13 apart, which is the same amount that 41 and 28 are apart."
- "I got 13 too, but I pictured it on a number line and counted up from 28 to 41."

We have also observed Number Talks in middle school, where this technique is used to find 40% of 250 or to solve for x when $3x - 24 = 21$. Students are generally engaged, and they are communicating mathematically and learning to value alternative approaches to doing mathematics and communicating and thinking mathematically.

Too often, however, in our classroom observations and discussions with math coaches, this magic of Number Talks is encountered only as part of classroom openers and fails to be transferred to mainstream or core mathematics instruction in the form of a consistent student-engaged Math Talk Classroom. Moreover, in our experiences, Number Talks tend to gradually disappear as students progress into middle school, and they are very rare in high school mathematics classrooms. We also are concerned that the traditional approaches to Number Talks entail the teacher being the recorder and not the students presenting their work. Moreover, the adherence to only mental mathematical approaches limits the inclusion of pictures and other critical representations and the difficulty of problems.

Given these common observations and concerns, our hope is to stimulate an expansion of the Number Talk technique throughout the grades and into high school, to broaden the use of Number Talk techniques into core lesson instruction, and to relax some of the strictures considered by some to be "the right way" to do Number Talks so that students can more frequently do their own recording and, when appropriate, can use whiteboards or pencil and paper to support their thinking, and can use multiple representations.

FROM NUMBER TALKS TO MATH TALK CLASSROOMS

So we are clear on what we envision in high-quality, equitable mathematics instruction, we offer the following definitions.

- A **Number Talk** is a strategy, often embedded at the beginning of a mathematics lesson, that elicits students' mental approaches to straightforward numerical exercises and that enables teachers to engage students in discussions of the connections among different approaches, errors, and misconceptions (Humphreys & Parker, 2015; Parrish, 2010; Parrish & Dominick, 2016; Parker & Humphreys, 2018). In so doing, Number Talks build mathematical agency and strengthen the focus on thinking, as opposed to regurgitation, in learning mathematics.

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- A **Math Talk Classroom** is a cultivated classroom culture and set of norms characterized by a consistent focus on student representation and discourse about student thinking and justification that is expected throughout a mathematics lesson (Hufferd-Ackles et al., 2004, 2015).

Given the challenges of launching and implementing a Math Talk Classroom that reduces reliance on activity sheets and minimizes memorizing mindless rules and repetitive practice, teachers have found that Number Talks are convenient and accessible entry points to the transition to problem-based representing and solving, alternative approaches, multiple representations, and a focus on student thinking. Table 1 summarizes the gradual shift from teacher-controlled instruction (Level 0) to far more student-centered instruction (Level 3) that characterizes our vision of the Math Talk Classroom (Hufferd-Ackles et al., 2004, 2015).

Our claim is that Number Talks are a great way to learn to elicit and discuss student thinking, but their impact is seriously compromised when they do not carry over into Math Talk Classrooms. In Table 1 we show levels in such learning as teachers progress from traditional teacher showing/talking to supporting student-to-student talking. These levels were identified in classrooms of teachers who were successful in making this transition. The changes in the teacher role can help teachers transition to higher levels as they support their students to learn to interact with each other instead of just sitting and listening to the teacher. Changes in questioning help everyone change from focusing only on answers to focusing on how answers were obtained and explaining methods. Questions also change from coming only from the teacher to coming from students. As the Math Talk Classroom develops, some of these questions will be questions students have, whereas others will be questions students pose to help other students notice or understand some concept. Explaining thinking develops as the teacher asks more probing questions and helps students by extending their explanations. A simple "Can you convince us?" and "Who can convince us in a different way?" are powerful follow-ups. Students learn that they are expected to explain as fully as they can, and other students listen carefully and also help with extending explanations.

Table 2 summarizes this transition from Number Talks to Math Talk Classrooms by presenting aspects of Number Talks (Humphreys & Parker, 2015; Parrish, 2010; Parrish & Dominick, 2016; Parker & Humphreys, 2018) and how they can be expanded to create Math Talk Classrooms (Hufferd-Ackles et al., 2004, 2015).

NUMBER TALKS AND THEIR EXPANSION ARE JUST AS VALUABLE IN MIDDLE AND HIGH SCHOOL

There is no reason to limit Number Talks and their extension into Math Talk Classrooms to elementary school. In fact, all the reasons we have described for shifting Number Talks and using them to drive more effective instruction throughout a lesson also apply to middle school and high school. We would argue that the typical gradual reduction in the use of representations and student explanations, along with the increased attention to just getting correct answers, as students progress through Grades 6 to 12 make these representing and explaining practices essential aspects of ratcheting up student learning of such critical aspects of the middle and secondary mathematics curriculum as proportional reasoning, algebra, geometry, and statistics.

Consider reducing the number of exercises on a typical practice activity sheet and launching a geometry lesson with the question: "About how much is the volume of a cylindrical metal can 10 cm tall and with a diameter of 6 cm?" Like an effective Number Talk, student discussion raises key vocabulary that students either already know or need to know, including *volume*, *diameter*, *centimeter*, *radius*, and *cubic units*. Beginning with making an estimate eliminates the need to start with, and simply apply, a formula and stimulates discussion, drawn from student ideas, of what is meant by volume; how length, area, and volume are related and linked to dimension; and how what we know about rectangular solids can help us consider cylinders. In other words, building from Number Talks and creating Math Talk Classrooms can result in far deeper understanding than the mere mindless memorization of $V = \pi r^2 h$. And even when some students already know and use the formula, the follow-up questions, "Why does that work, and what do the variables mean?" enrich the discussion for all students.

Just as extending the strengths and power of Number Talks to core instruction deepens the focus on understanding the key ideas of elementary mathematics, class discussions, often before formal instruction, can strengthen the teaching and learning of secondary mathematics. A lesson launch in middle or high school that asks students to find the value of x, explain their

Table 1 Levels and Components of Math Talk Learning Communities

	Teacher Role	Questioning	Explaining Mathematical Thinking	Mathematical Representations	Building Student Responsibility Within the Community
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher's questions focus on correctness. Students provide short answer-focused responses. Teacher may tell answers.	Representations are missing or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
Level 1	Teacher encourages sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher's questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to depict their mathematical thinking.	Students feel their ideas are accepted by the classroom community. They begin to listen to each other supportively and to restate in their own words what another student said.
Level 2	Teacher facilitates conversation between students and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so others are able to follow their mathematical thinking.	Students believe they are math learners and that their ideas and the ideas of classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.	Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.	Students believe they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same.

Note. Reprinted with permission from Hufferd-Ackles et al. (2015).

Table 2 Building on the Foundation of Number Talks

Aspects of Number Talks	Expanding Number Talks to Math Talk Classrooms
Number Talks are often used as lesson openers, separate from, and before, the core part of the lesson to which the powerful strategies of Number Talks are not transferred.	Students need good teaching every day throughout their mathematics class. All mathematics classes should include students explaining their thinking. Number Talks can help teachers learn how to elicit and support student thinking. But then it is vital that teachers elicit and support student thinking during the rest of grade-level mathematics lessons.
Many teachers incorporate student justification and discussion into their Number Talks, but fail to adapt regular mathematics teaching to include similar and consistent justification and discussion.	A key aspect of the Common Core State Standards for Mathematics is the third of the Standards for Mathematical Practice (SMP 3): "Construct viable arguments and critique the reasoning of others." Number Talks are perfect vehicles for launching and practicing this focus on justification and argument, but it is vital that teachers use such prompts as "Why is that?", "Can you explain your reasoning?", "Can you convince us?", and "How did your brain picture that?" to expand these Number Talk elements to mainstream instruction.
As often recommended, many teachers limit students to only mental methods when conducting Number Talks and rarely expand the Number Talk to include visual representations that can support solutions and enhance instruction. Giving students access to whiteboards, markers, or pencils can reduce student thinking into just using an algorithm, but broadening Number Talks beyond mental mathematics can be a safe place for students to practice the use, demonstration, and defense of multiple representations.	Many state and district standards ask teachers and students to use visual representations because they support meaning-making and learning. Not allowing students to use visual supports violates those standards and reduces the opportunity to understand those standards. Because this is what is expected in mainstream instruction, these practices ought to extend, when appropriate, to Number Talks as part of Math Talk Classrooms. In addition, restricting students to mental solutions can reduce the difficulty of problems that can be used to a level that might not measure up in complexity to what is done in mainstream instruction. This is a particular concern when teaching English language learners because many such students are helped to solve and to explain by visual supports that make the mathematical language and symbols meaningful. The Math Talk Classroom benefits greatly from presentation, use, and discussion of such visual representations.
Teachers write and draw the methods that students describe orally so that they are visible to the entire class.	Students may learn from this that only the teacher can write or draw to show thinking. Students need to draw their own methods. Other students and the teacher can help to correct and extend such drawings.
Number Talks are often stimulated by naked computation exercises.	Teachers can open discussion of topics at any time within the regular mathematics class by providing students with diagrams, objects, exercises, tasks, or problems; by eliciting student methods; and by leading a discussion that compares and relates the different methods, insights, and approaches. All of a mathematics class needs to open problems to student thinking and discussing, as is typically done with Number Talks.
Parker and Humphreys (2018) explain that they ask teachers to do all of the recording of student thinking for three main reasons: "(1) clear communication, (2) accurate representation, and (3) precise mathematical notation" (p. 87).	Teachers can model how to represent student methods sometimes, but students need to learn how to do accurate recording of their thinking. Students also need to be able to make mistakes and then edit and improve their representations and explanations. Discussing student written solutions with mathematics drawings allows all of this to happen and supports improving written representations to take place by all students. If students are recording their own thinking in drawings related to mathematical notation, often individually or in pairs on whiteboards, the teacher is freed up to think deeply about the student method and has more cognitive time to make the difficult teaching decisions about what to elicit and how to focus discussion. The explaining student, other students, and the teacher can raise questions and extend thinking about a given drawing and method. Thus, everyone learns about these three crucial aspects of mathematical representing.

thinking, and describe how they know they are correct using simple straightforward tasks like the following can be very productive.

- In the proportion $3/8 = x/56$, what must the value of x be? Now find your answer another way.
- What is the value of x that makes the equation $3x + 19 = 46$ true? Now show a different way to get that value.
- A cube with side length x units long has a volume of 1,000 cubic units. What is the value of x, and how can you prove it?

It should be clear that orchestrating such discussions as part of Math Talk Classrooms serves to diagnose a range of prior knowledge that is needed for learning a skill or concept as well as inform the scope of remediation or reteaching that may be necessary. Even when students use a rule or procedure and arrive at a correct answer, these instructional strategies open windows on why rules work, show alternatives to one-right-way approaches, and demonstrate the power of deeper understanding to offset forgetting the rule.

SOCIAL NORMS FOR A MATH TALK COMMUNITY

To open up mathematics classes to students expressing their own solution methods, teachers need to extend the social norms they create for Number Talks to regular mathematics class. Students need to be valued and supported as they describe or explain a solution method. New topics need to begin by eliciting what students know and think, stimulated by open-ended questions such as "What do you notice?" and "What do you wonder?" Students also need to develop or learn a common visual language to see and understand the mathematical concepts involved. Students can contribute their own ways of seeing or showing a concept, but other appropriate visual representations are also powerful. Having a shared visual language facilitates communication, especially if the teacher helps students to understand and demonstrate new mathematical language.

Mathematical representations become central in many solutions and explanations. Students may invent these, or they may come from the mathematics program or the teacher as a common visual language to support thinking and communicating. Labeling and editing drawings increase as students communicate more fully through drawings as well as through oral language. In classrooms with many students learning English, drawings serve as referents for learning new mathematical language and communicating about mathematical concepts. The final column in Table 1 about building student responsibility grows from all the other columns. Having their voices heard by the teacher and classmates, and being supported to explain by drawings and classmates and the teacher, gradually creates a nurturing Math Talk Community in which students increasingly take responsibility for their own learning and for the learning of their classmates. Just as drawings and other representations play a key role in core instruction, they should play a key role in Number Talks as well.

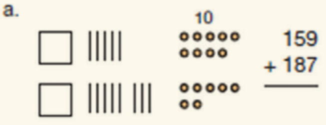
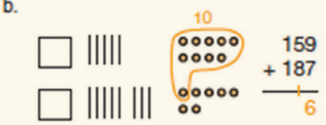
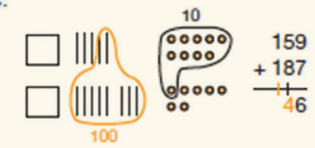
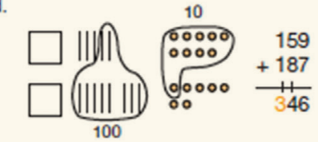
Number Talks, when done well, serve an important social and emotional goal: They help students believe in themselves as problem solvers. When teachers help each student be seen as a capable explainer, Number Talks can also help students believe in their classmates as problem solvers. Teachers can open their classrooms during regular mathematics lessons by moving over time through the levels in Table 1 while students use concrete materials or mathematical drawings and verbal justifications to show and explain mathematical concepts. Then, every mathematics class can sound like a Number Talk as students explain their thinking. Number Talks can also be diagnostic of where students are. Then, teachers can plan a combination of lessons focused on concepts that need understanding and grade-level goals, all with objects or drawings as learning supports.

EXTENDING NUMBER TALK STRATEGIES TO MATH TALK CLASSROOMS

Often, students will have a mathematical drawing and a written method relating to that drawing. This is consistent with important Common Core State Standards and other high-level standards. For example, 2.NBT.7 says: "Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method" (National Governors Center for Best Practices & Council of Chief State School Officers, 2010, p. 19). Figure 1 shows a Grade 2 student explaining a three-digit addition problem and then classmates asking questions. The explainer needs to stand to the side of the drawing and use a pointer to focus the attention of listening students and the teacher on the parts of the drawing or method the explainer is mentioning at that moment. Accompanying the verbal explaining with such pointing allows the explainer to connect the mathematical drawing to the written method. The questions of classmates allow parts of the explanation that were unclear to be clarified or extended.

Here is a comparable middle school example to demonstrate the power of Math Talk Classrooms that expand on Number Talks. Consider this Grade 7 Number Talk: Solve for x in the proportion $12/20 = x/35$. When it is presented as a naked proportion, we know that most

Figure 1 Drawings and Text for a Student Explanation of Generalizable and Accessible New-Groups-Below Method (NCTM, 2011, p. 87).

<table>
<tr><th>Math Drawing and Problem</th><th>Explanation Using Place-Value Language About Hundreds, Tens, and Ones</th></tr>
<tr><td>a. 
</td><td>I drew one hundred, five tens, and nine ones to show one hundred fifty nine, and here below it I drew one hundred, eight tens, and seven ones for one hundred eighty seven. I put the ones below the ones, the tens below the tens, and the hundreds below the hundreds so I could add them easily.</td></tr>
<tr><td>b. 
</td><td>See here in my drawing, nine ones need one more one from the seven to make ten ones that I circled here, and I wrote 10. That leaves six ones here. With the numbers the seven gives one to the nine to make ten that I write over here in the tens column, see one ten. And I write six ones here in the ones column.</td></tr>
<tr><td>c. 
</td><td>With the tens, I start with eight because it is more than five so it is easier. I get two tens from five tens to make ten tens, see here, and I write one hundred here to remind me that the ten tens make one hundred. There are three tens left in the five tens and I have one more ten from my ones (see here in my drawing and the one ten at the bottom of the tens column). That makes four tens and the one hundred. So in my problem I write the one hundred below in the hundreds column and the four tens in the tens column.</td></tr>
<tr><td>d. 
</td><td>There are three hundreds, two in the original numbers I'm adding and one new hundred from the ten tens. I write three hundreds here in the hundreds column. Are there any questions? Yes, Stephanie.</td></tr>
<tr><th>Student Question</th><th>Explainer Answer</th></tr>
<tr><td>Stephanie: For the tens, you never said fourteen tens as the total of the tens. Why not?</td><td>Because when I'm making ten tens, I just can write that one hundred over here with the hundreds and just think about how many tens I need to write. But I can think eight tens and five tens is thirteen tens and one more ten is fourteen tens, so that is one hundred and four tens. You can do it either way. (Aki)</td></tr>
<tr><td>Aki: Do you still need to make the drawings or did you just make them so you could explain better?</td><td>I don't have to make the drawings, but I can explain better with a drawing because you can see the hundreds, tens, and ones so well. (Jorge)</td></tr>
<tr><td>Jorge: Do you do make-a-ten in your head or just know those answers?</td><td>I just know all of the nine totals because of the pattern: the ones number in the teen number is one less that the number added to nine because it has to give one to nine to make ten. So nine plus seven is sixteen. I just know that pattern super fast. For eight plus five, I do make-a-ten fast, sort of just thinking five minus two is three, so thirteen. (Sam)</td></tr>
<tr><td>Sam: I know five and eight is thirteen, so why did you write a four in the tens column, Karen?</td><td>Because I had one more ten from the ones. See here in the drawing: nine ones and one one from the seven ones make ten ones. I wrote 10 here to remind me, and here in the problem I wrote the new one ten below where I can add it in after I find thirteen. You have to write your new one ten big enough to be sure you see it.</td></tr>
<tr><td>Sam: Oh yes, I see it now. I can see the new one ten when I write it, but I couldn't see yours.</td><td>OK, thanks. I'll write it bigger next time so everyone can see it.</td></tr>
</table>

students will try cross multiplication, run into trouble with the mental mathematics, and give up without a calculator or pencil and paper. We also know that some students will wisely reduce 12/20 to 3/5 and understand that since 35 is 5 times 7, the answer must be 3 times 7, or 21. There may be a few more approaches but rarely ones that move students away from the cross-multiplication algorithm, thereby severely limiting the value of such a Number Talk.

Now consider the Math Talk Classroom where "What was your answer, and how did you get it?" is expanded to "Who did it differently?" and "How did you picture it?" Even better, consider the Grade 7 classroom presented with the following task: It costs $12 to buy 20 bananas. Sarah needs 35 bananas. How much will Sarah have to pay for the bananas she needs?

The problem situation generates a need to consider which operation or operations to use instead of just using a rule and manipulating the given numbers. The fact that the problem is posed without a proportion opens doors to many different approaches that enliven the discussion and deepen understanding. The expectation that work be shown on a whiteboard further opens the door to alternative approaches, multiple representations, and extensive discussion about the connections between and among the approaches and the representations. For example, consider the explanation and Math Talk that emerges from whiteboard work like the following:

- Robert's whiteboard explains:
  First I got 20 bananas for $12, then half that amount or 10 bananas for $6 and half again or 5 bananas for $3. 12 + 6 + 3 equals $21.
- Max's whiteboard shows:
  $6 lllll lllll
  $6 lllll lllll
  $6 lllll lllll
  $3 lllll
  So $21 for 35 bananas as shown in my picture.
- Maria's whiteboard shows a table:

Money	Bananas
$3	5
$6	10
$9	15
$12	20
$15	25
$18	30
$21	35

  My table shows that 35 bananas will cost $21.
- Jamar's whiteboard shows a different picture from Max's:
  $3 = lllll
  $3 = lllll
  If you need 10 bananas, you pay $6 and for 5 bananas, you pay $3.
  So 35 bananas is three groups of 10 plus one group of 5.
  That's 3 × $6 plus 1 × $3 or $21.
- Donelle's whiteboard explains:
  I divided 20 by 4 and got 5.
  I divided 12 by 4 and got 3.
  So 5 bananas cost $3. I carried out the rate and got 35 bananas cost $21.

B	5	10	15	20	25	30	35
$	$3	$6	$9	$12	$15	$18	$21

- Pat's whiteboard shows:
  12/20 = x/35 so 12 × 35 = 20x
  35 × 12 = 420 and 420 divided by 20 = 21.
- And finally Hector's whiteboard says:
  I did this by finding out what each banana costs by dividing 12 by 20 on a calculator.
  I got 0.6 which means that each banana costs 6/10 of a dollar or 60 cents.
  60 cents times 35—also done on my calculator—equals $21.

In a Math Talk Classroom, tasks like this with a focus on alternative approaches, multiple representations, explanations, and comparisons between and among solutions significantly expand Number Talks.

INCORPORATING POWERFUL REPRESENTATIONS INTO MATH TALK CLASSROOMS

Some mathematics drawings have been used successfully for many years by many students and have appeared in National Council of Teachers of Mathematics (NCTM) publications or national reports. We briefly overview these drawings to help teachers unfamiliar with them build their Math Talk Classroom. We include some of the knowledge about student thinking that we have learned from our decades in classrooms.

Drawings of hundreds, tens, and ones (see Figure 1) have been used for a long time (NCTM, 2009a, 2011; National Research Council, 2001). The use of five-groups in the ones and tens makes it possible to tell at a glance how many there are, reducing errors and helping observers check and understand the drawings. Five-groups support advanced single-digit methods like the make-a-ten methods shown in Figure 1: For 9 + 7, you can see that the 9 needs one more to make 10; take the 1 from the 7, leaving 6; 10 + 6 is 16. These drawings can be built up meaningfully by having children draw columns of 10 dots and then draw a ten-stick through the 10 dots. This ten-stick can then be abbreviated by just drawing a little stick, but students can still see the imaginary 10 dots on the stick and sometimes draw some of them in problem solving. So, 10 ten-sticks can be seen to be 10 tens (10 groups of 10 dots) and also 100 single dots, shown by a sketch of a square enclosing all of these invisible dots.

Multidigit multiplication and division have for a long time used area models in which multidigit lengths are written in their place value chunks along the sides of a rectangle, and the area is partitioned to show the different partial products made by these place values. Division can be shown as the known factor on the left side, and the unknown factor building up along the top as place-value chunks. Students develop different ways of recording written methods from these drawings, and these methods can be explained and compared (Common Core Standards Writing Team, 2015; Fuson & Beckmann, 2012–2013; NCTM, 2009b).

Fraction number lines are a mathematical tool that students must come to understand. Number lines are initially difficult because many students focus on the vertical marks on the lines or on the numbers written below those marks and are off by one (e.g., Izsák et al., 2008). Fraction bars are easier because you can see the unit lengths and can write unit lengths inside each part, so 1/4 + 1/4 + 1/4 can be seen to be 3/4, three of the unit fractions 1/4. The running totals of the unit fractions (1/4, 2/4, 3/4, 4/4, 5/4, etc.) can be written along the bottom of the bar, making that bottom line a number line. Or a number line can be drawn below the fraction bar and labeled so that the unit lengths are the same. Approaching number lines so that students focus on their unit lengths enables students to use them accurately and with understanding.

Number lines are not so useful for representing multidigit computation because the number line for 100 fits across an 8 1/2-by-11-inch page only with unit lengths of 1/16. These units are very small. Students cannot draw such number lines, and the units are difficult to work with. The Common Core does not require using number lines for computation. Standard 2.MD.2.6 asks students only to represent whole-number sums and differences within 100 on a number line diagram, that is, to show adding as having the length of one addend added onto the length of the other addend and to show subtracting as having the known addend within the total with both starting at zero. The open number line used in some approaches is not a number line diagram, which needs the same actual units its whole length. The open number line is a sketch that allows one to show parts and running totals above and below the sketch to relate them. But other ways of recording added-on numbers generalize to numbers larger than 100, so open number lines are unnecessary and are difficult for some students to use for three-digit numbers.

Mathematical drawings for decimals need to start with a one that is large enough to be partitioned into tenths and the tenths into hundredths. A meterstick marked in decimeters and centimeters is one useful tool. Also, a square representing one whole can be divided into tenths along each side, giving small squares inside that are hundredths. An important point that can reduce errors concerning place-value units is to emphasize that the symmetry is around the ones place and not around the decimal point. So, the tens and tenths are next to the ones place, and they are 10 times as many and 1/10 times as many as one. The decimal point makes some students ask where the "oneths" place is because they want a decimal place to mirror the ones place.

CONCLUDING THOUGHTS

Number Talks can be powerful tools for engaging students in sharing their thinking and capturing alternative approaches to finding solutions. However, when limited to elementary grade classrooms, when limited to mental mathematics strategies with little inclusion of representations, when reliant on teacher annotation, and when not carried over to core classroom mathematics instruction, the power of Number Talks is limited unnecessarily. Our hope is that the power of Number Talks is expanded to core instruction, to middle and high school, to incorporate representations, and is used to convert teacher-directed classrooms into Math Talk Classrooms that truly empower students.

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