

Karen C. Fuson

Research Shorter Publications for Teachers

- Fuson, K. C. (1991). Children's early counting: Saying the number-word sequence, counting objects, and understanding cardinality. In K. Durkin & B. Shire (Eds.), *Language and mathematical education* (pp. 27-39). Milton Keynes, GB: Open University Press.
- Fuson, K. C. & Kwon, Y. (1991). Chinese-based regular and European irregular systems of number words: The disadvantages for English-speaking children. In K. Durkin & B. Shire (Eds.), *Language and mathematical education* (pp. 211-226). Milton Keynes, GB: Open University Press.
- Fuson, K. C. (1992). Relationships between counting and cardinality from age 2 to age 8. In J. Bideaud, J. P. Fischer, & C. Meljac (Eds.), *Pathways to number* (pp. 127-149). Hillsdale, NJ: Lawrence Erlbaum Associates. [This is a book of invited chapters celebrating the fiftieth anniversary of the publication of Piaget's book on number. Published in French in *Les chemins du nombre* (pp. 159-179). Villeneuve d'Ascq, France: Presses Universitaires de Lille.]
- Fuson, K. C. & Kwon, Y. (1992). Learning addition and subtraction: Effects of number words and other cultural tools. In J. Bideaud & C. Meljac (Eds.), *Pathways to number* (283-306). Hillsdale, NJ: Lawrence Erlbaum Associates. [This is a book of invited chapters celebrating the fiftieth anniversary of the publication of Piaget's book on number. Published in French in *Les chemins du nombre* (pp. 351-374). Villeneuve d'Ascq, France: Presses Universitaires de Lille.]
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- Fuson, K. C., Lo Cicero, A., Hudson, K., & Smith, S. T. (1997). Snapshots across two years in the life of an urban Latino classroom. In Hiebert, J., Carpenter, T., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., Human, P., *Making sense: Teaching and learning mathematics with understanding* (pp. 129-159). Portsmouth, NH: Heinemann.
- Fuson, K. C., Sherin, B., & Smith, S. T. (1998). A Vygotskian action-research model for developing and assessing conceptual models and instructional materials inter-actively. In S. Berenson, K. Dawkins, M. Blanton, W. Coulombe, J. Kolb, K. Norwood, & L. Stiff (Eds.), *Proceedings of the Twentieth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 2* (pp. 541-548). Raleigh, NC: North Carolina State University.
- Ron, M. P. (1998). My parent taught me this way: The European/Latino multidigit subtraction algorithm and confusions with a U.S. algorithm. In L. J. Morrow & M. J. Kenney (Eds.). *The teaching and learning of*

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- De La Cruz, Y. (1999). Reversing the trend: Latino families in real partnerships with schools. *Teaching Children Mathematics*, 5 (5), pp. 296-300.
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- Fuson, K. C. & Lo Cicero, A. M. (2000). El Mercado in Latino primary math classrooms. In M. L. Fernandez (Ed.), *Proceedings of the Twenty-Second Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 2* (p. 453). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
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- Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2015). Describing levels and components of a Math-Talk Learning Community. In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Volume 1: Useful and usable research related to core mathematical practices* (pp. 125-134). Reston, VA: NCTM.
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- Fuson, K. C. (2019). Overcoming errors in fraction computation by emphasizing unit fractions, length drawings, and student explanations. *Universal Journal of Educational Research*, 7(8): 1663-1678, 2019 DOI: 10.13189/ujer.2019.070805
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Longer Research Publications

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- Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag. [Chapter 11: An overview of changes in children's number word concepts from age 2 through 8]
- Fuson, K. C., Stigler, J., Bartsch, K. (1988). Grade placement of addition and subtraction topics in China, Japan, the Soviet Union, Taiwan, and the United States. *Journal for Research in Mathematics Education*, 19, 449-458.
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